Chapter 3

Order flow, Liquidity and Securities Price Dynamics
3.8 Exercises

2. Bid-ask spread and insider trading. A small risky company’s stock is worth either $10 \( (v^L) \) or $20 \( (v^H) \) with probability \( \frac{1}{2} \) each \( (\theta = 1 - \theta = 0.5) \).
   
   a. Compute the bid and ask prices set by risk neutral competitive market makers in the absence of informed trading.
   
   b. Compute the bid and ask prices set by risk neutral competitive market makers when they expect one in ten of trade initiators to be informed (to know the stock’s true value) and to trade as profit maximizers, while the other nine out of ten are uninformed and buy or sell with equal probability. Assume that all transactions are of the same size.
   
   c. Compute the average trading cost to an uninformed trader and the average gain to an informed one, assuming a unit trade size in both cases.
   
   d. Do you agree with the following statement? “Insider trading does not harm most market participants: it harms only those who are unlucky enough to trade with an insider.” Why? Refer to the example in this exercise to illustrate your argument.

4. Endogenous information acquisition by insiders in the Glosten-Milgrom model. Consider the one-period Glosten-Milgrom model, where the security’s true value \( v \) can be high \( (v^H) \) or low \( (v^L) \) with probability \( \frac{1}{2} \) each. Market makers are competitive and risk neutral, and do not know \( v \). In each period, a single trader comes to the market: with probability \( 1 - \pi \), he is a “noise trader,” who buys or sells one unit with probability \( \frac{1}{2} \) each; with probability \( \pi \) he is a “potential insider,” who learns the security’s true value \( v \) if he pays a cost \( c \), in which case he will trade on his information to make a profit. (If he does not elect to acquire information, he does not trade.)
a. Compute the bid and ask prices and the bid-ask spread that market makers set, assuming that they believe the insider will acquire information with some given probability \( \varphi \in [0, 1] \)?

b. Given these prices, determine the trading profit of an insider who has decided to acquire information.

c. Determine the condition on the cost parameter \( c \) in terms of \( \pi \) and \( v^H - v^L \) under which the potential insider will never choose to acquire information, even in the most auspicious situation in which market makers do not expect to face any insider trading (i.e., \( \varphi = 0 \)).

d. Determine the condition on the cost parameter \( c \) in terms of \( \pi \) and \( v^H - v^L \) under which the potential insider will always choose to acquire information, even in the least auspicious situation in which market makers expect to face insider trading with probability \( \pi \) (i.e., \( \varphi = 1 \)).

e. Now consider intermediate values of \( c \) for which neither of the conditions determined under (c) and (d) is satisfied. Determine the value of \( \varphi \) that will make the potential insider indifferent between acquiring and not acquiring information. This endogenous value of \( \varphi \) describes a “mixed-strategy equilibrium” in which the insider randomizes between the two options. How does this value of \( \varphi \) depend on the parameters of the model? Explain your results intuitively.

f. Characterize the equilibrium in the three ranges of the values of \( c \) considered in (c), (d), and (e). Plot the bid-ask spread and the potential insider’s net expected profit as a function of the cost \( c \) of acquiring information. Explain your graph intuitively.
3.9 Solutions

Exercise 2:

a. Under these assumptions, there is no bid-ask spread since the probability of informed trading, $\pi$, is equal to 0. Ask and bid prices are both equal to $\mu$, where 
$$\mu = \frac{(v^H + v^L)}{2} = \frac{(10 + 20)}{2} = 15.$$ 

b. Under these assumptions the probability of informed trading is $\pi = 1/10$, that can be used in the general expressions for bid and ask prices provided in the Glosten-Milgrom model:
$$a = \mu + \frac{\pi\theta(1 - \theta)}{\pi\theta + (1 - \pi)} \frac{(v^H - v^L)}{2} = 15 + \frac{0.1 \cdot \frac{1}{2} \cdot \frac{1}{2}}{0.1 \cdot \frac{1}{2} + 0.9 \cdot \frac{1}{2}} (20 - 10) = 15 + 0.5 = 15.5,$$
and
$$b = \mu - \frac{\pi\theta(1 - \theta)}{\pi\theta + (1 - \pi)} \frac{(v^H - v^L)}{2} = 15 - \frac{0.1 \cdot \frac{1}{2} \cdot \frac{1}{2}}{0.1 \cdot \frac{1}{2} + 0.9 \cdot \frac{1}{2}} (20 - 10) = 15 - 0.5 = 14.5.$$ 

Hence the absolute bid-ask spread is
$$S = a - b = 1.$$ 

c. An uninformed trader who trades a unit of the security pays a trading cost of $0.5$ whilst the expected gain for an informed trader is $\$[(20 - 15.5) \cdot 0.5 + (14.5 - 10) \cdot 0.5] = 4.5$. The market maker is operating at zero profits since his expected profits are $\$[(0.5) \cdot 0.9 - (4.5) \cdot 0.1] = 0.45 - 0.45 = 0.$$

d. Insider traders do not harm only those who trade with them. As shown at point (c) market makers recoup the losses they make because of informed trading by charging the bid-ask spread to all their customers. Basically uninformed traders pay the cost arising from informed trading, even if they do not trade directly with them.

Exercise 4:
a. Market makers know that the probability of receiving an order from an uninformed trader is $1 - \pi$, the probability of receiving an order from an informed trader is $\pi \varphi$, and the probability of receiving no order (which occurs when the potentially informed trader did not discover the information) is $\pi(1 - \varphi)$. Hence, the ask price is

$$a = \frac{\pi \varphi v^H + (1 - \pi) \frac{v^H + v^L}{2}}{\pi \varphi + (1 - \pi)}$$

and the bid price is

$$b = \frac{\pi \varphi v^L + (1 - \pi) \frac{v^H + v^L}{2}}{\pi \varphi + (1 - \pi)}.$$  

Hence the absolute bid-ask spread is

$$S = a - b = \frac{\pi \varphi}{\pi \varphi + (1 - \pi)} (v^H - v^L),$$  

which is increasing in $\varphi$: if potential insiders discover the information with greater likelihood, there will be more adverse selection and therefore a larger spread.

b. An insider who observes $v = v^H$ and buys at the ask price makes a profit of

$$v^H - a = v^H - \frac{\pi \varphi v^H + (1 - \pi) \frac{v^H + v^L}{2}}{\pi \varphi + (1 - \pi)} = \frac{1 - \pi}{2 \pi \varphi + (1 - \pi)} (v^H - v^L).$$

Symmetrically, if he observes $v = v^L$ and sells at the bid price makes the same profit:

$$b - v^L = \frac{\pi \varphi v^H + (1 - \pi) \frac{v^H + v^L}{2}}{\pi \varphi + (1 - \pi)} - v^L = \frac{1 - \pi}{2 \pi \varphi + (1 - \pi)} (v^H - v^L).$$

If the potential insider invests in information collection, he earns the first amount half of the time (when he observes $v = v^H$) and the second (identical) amount the other half of the time (when he observes $v = v^L$), and pays the cost $c$. Hence, his net expected profit is

$$\frac{1}{2} \frac{1 - \pi}{\pi \varphi + (1 - \pi)} (v^H - v^L) - c.$$  

(3.64)
c. No information is collected if
\[ c > \frac{1}{2} \frac{1 - \pi}{\pi \varphi + (1 - \pi)}(v^H - v^L). \]  
(3.65)
Since the right-hand side of (3.65) is decreasing in \( \varphi \), it is largest for \( \varphi = 0 \). Hence, if it holds for \( \varphi = 0 \), it will also hold for \( \varphi > 0 \). Hence, no information will be collected if
\[ c > \frac{1}{2}(v^H - v^L), \]  
(3.66)
which is condition (3.65) for \( \varphi = 0 \).

d. By the same argument presented under (c), the right-hand side of condition (3.65) is smallest for \( \varphi = 1 \). Hence if this condition is violated for \( \varphi = 1 \), it will be violated for any \( \varphi \in [0, 1] \). Hence, information collection is always profitable if
\[ c < \frac{1}{2}(1 - \pi)(v^H - v^L). \]  
(3.67)
which is condition (3.65) for \( \varphi = 1 \).

e. The potential insider is indifferent between collecting and not collecting information if condition (3.65) holds with equality instead of inequality:
\[ c = \frac{1}{2} \frac{1 - \pi}{\pi \varphi + (1 - \pi)}(v^H - v^L), \]
that is, if
\[ \varphi^* = \frac{1 - \pi}{\pi} \left( \frac{1}{2} \frac{v^H - v^L}{c} - 1 \right). \]  
(3.68)
f. If condition (3.67) holds, then \( \varphi = 1 \) and \( S = \pi(v^H - v^L) \).
If condition (3.66) holds, then \( \varphi = 0 \) and \( S = 0 \).
In the intermediate region where
\[ \frac{1}{2}(1 - \pi)(v^H - v^L) \leq c \leq \frac{1}{2}(v^H - v^L), \]
the bid-ask is given by the general expression (3.63) derived under point (i), which upon substituting from (3.68) becomes

\[ S = 1 - \frac{2c}{v^H - v^L}. \]  

This expression is decreasing in the information collection cost \( c \): intuitively, a higher cost discourages information collection (lowers \( \varphi^* \)), thus reducing the probability of informed trading (lowers \( \pi^* \)) and the implied adverse selection problems. The bid-ask spread in this intermediate region is still increasing in fundamental volatility \( v^H - v^L \): intuitively, greater volatility raises the profits from informed trading (as shown at point (b)) and thereby encourages information collection (lowers \( \varphi^* \)), thus increasing the probability of informed trading (lowers \( \pi^* \)) and the implied adverse selection problems.

Figure 3.1 puts together the previous results in a single graph: for low values of the information collection cost \( c \) potential insiders always seek information and the bid-ask spread \( S \) is highest (at \( S = \pi(v^H - v^L) \)); for intermediate values of this cost, it decreases linearly in \( c \) as shown by (3.69); for high values of the cost \( c \), no information is collected and the bid-ask spread \( S \) becomes zero.

Figure 3.2 instead plots the net profit of the informed trader. For low values of the information collection cost \( c \), this profit is obtained by setting \( \varphi = 1 \) in expression (3.64). For intermediate values, the profit is zero, because the potential insider chooses \( \varphi \) precisely to be indifferent between being informed and uninformed: this can be verified by substituting \( \varphi^* \) from (3.68) in expression (3.64).
Figure 3.1: Bid-ask spread $S$ and information collection cost $c$

Figure 3.2: Informed trader’s net profit and information collection cost $c$