

# Chapter 8

## Benefit/Cost Ratios and Other Measures

### BENEFIT COST

#### 8-1

Rash, Riley, Reed, and Rogers Consulting has a contract to design a major highway project that will provide service from Memphis to Tunica, Mississippi. R<sup>4</sup> has been requested to provide an estimated B/C ratio for the project. Relevant data are:

Initial cost	\$20,750,000
Right of way maintenance	550,000
Resurfacing (every 8 years)	10% of first cost
Shoulder grading and re-work (every 6 years)	750,000
Average number of road users per year	2,950,000
Average time savings value per road user	\$2

Determine the B/C ratio if  $i = 8\%$ .

#### **Solution**

$$\begin{aligned} AW_{\text{BENEFITS}} &= 2,950,000 \times \$2 = \$5,840,000 \\ AW_{\text{COSTS}} &= 20,750,000(A/P, 8\%, \infty) + 550,000 + .10(20,750,000)(A/F, 8\%, 8) \\ &\quad + 750,000(A/F, 8\%, 6) = \$2,507,275 \end{aligned}$$

$$B/C = \frac{AW_{\text{BENEFITS}}}{AW_{\text{COSTS}}} = \frac{5,840,000}{2,507,275} = 2.33$$

#### 8-2

A proposed bridge on the interstate highway system is being considered at the cost of \$2 million. It is expected that the bridge will last 20 years. The federal and state governments will pay these construction costs. Operation and maintenance costs are estimated to be \$180,000 per year. Benefits to the public are estimated to be \$900,000 per year. The building of the bridge will result in an estimated cost of \$250,000 per year to the general public. The project requires a 10% return. Determine the B/C ratio for the project. State any assumption made about benefits or costs.

#### **Solution**

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\$250,000 cost to general public is disbenefit.

$$AW_{\text{BENEFITS}} = 900,000 - 250,000 = \$650,000$$

$$AW_{\text{COSTS}} = 2,000,000(A/P, 10\%, 20) + 180,000 = \$415,000$$

$$B/C = \frac{AW_{\text{BENEFITS}}}{AW_{\text{COSTS}}} = \frac{650,000}{415,000} = 1.57$$

### 8-3

The town of Podunk is considering building a new downtown parking lot. The land will cost \$25,000 and the construction cost of the lot is estimated to be \$150,000. Each year costs associated with the lot are estimated to be \$17,500. The income from the lot is estimated to be \$18,000 the first year and increase by \$3,500 each year for the twelve year expected life of the lot. Determine the B/C ratio if Podunk uses a cost of money of 4%.

#### Solution

$$PW_{\text{BENEFITS}} = 18,000(P/A, 4\%, 12) + 3,500(P/G, 4\%, 12) = \$334,298$$

$$PW_{\text{COSTS}} = 175,000 + 17,500(P/A, 4\%, 12) = 339,238$$

$$B/C = \frac{PW_{\text{BENEFITS}}}{PW_{\text{COSTS}}} = \frac{334,298}{339,238} = 0.99$$

### 8-4

Tires-R-Us is considering the purchase of new tire balancing equipment. The machine will cost \$12,699 and have an annual savings of \$1,500 with a salvage value at the end of 12 years of \$250. If the MARR is 6%, use B/C analysis to determine whether or not the equipment should be purchased.

#### Solution

$$PW_{\text{BENEFITS}} = \$1,500(P/A, 6\%, 12) + \$250(P/F, 6\%, 12) = \$12,700.25$$

$$PW_{\text{COSTS}} = \$12,699$$

$$B/C = 12,700/12,699 = 1.00$$

Conclusion: Yes, the machine should be purchased

### 8-5

Dunkin City wants to build a new bypass between two major roads that will cut travel time for commuters. The road will cost \$14,000,000 and save 17,500 people \$100/yr in gas. The road will need to be resurfaced every year at a cost of \$7,500. The road is expected to be used for 20 years. Determine if Dunkin City should build the road using B/C analysis. The cost of money is 8%.

#### Solution

$$PW \text{ of Costs} = 14,000,000 + 250,000(P/A, 8\%, 20) = \$16,454,500$$

$$PW \text{ of Benefits} = (17,500)(100)(P/A, 8\%, 20) = \$17,181,500$$

$$B/C = 17,181,500/16,454,500 = 1.04$$

Conclusion: Yes, Dunkin City should build the bypass

## FUTURE WORTH

### 8-6

Lucky Lindy has just won \$20,000 and wants to invest it for 12 years. There are three plans available to her.

- a) A savings account that pays 3¾% per year, compounded daily.
- b) A money market certificate that pays 6¾% per year, compounded semiannually.
- c) An investment account that based on past experience is likely to pay 8½% per year.

If Lindy does not withdraw the interest, how much will be in each of the three investment plans at the end of 12 years?

### Solution

$$a) \quad F = P(1 + i)^n$$

$$i_{eff} = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{.0375}{365}\right)^{365} - 1 = 3.82\%$$

$$FW = \$20,000(1 + .0382)^{12} = \$31,361.89$$

$$b) \quad i_{eff} \left(1 + \frac{.0675}{2}\right)^2 - 1 = 6.86\%$$

$$FW = \$20,000(1 + .0686)^{12} = \$44,341.67$$

$$c) \quad FW = \$20,000(1 + 0.115)^{12} = \$73,846.24$$

Choose plan C since this plan yields the highest return at the end of 12 years.

### 8-7

Bee-Low Mining Inc. must purchase a new coring machine that costs \$30,000 and is expected to last 12 years, with a salvage value of \$3,000. The annual operating expenses are expected to be \$9,000 the first year and increase by \$200 each year thereafter. The annual income is expected to be \$12,000 per year. If Bee-Low's MARR is 10%, determine the NFW of the machine purchase.

### Solution

$$NFW = -30,000(F/A, 10\%, 12) - [9,000 + 200(A/G, 10\%, 12)](F/A, 10\%, 12)$$

$$+ 12,000(F/A, 10\%, 12) + 3,000$$

$$= \$-45,754$$

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**8-8**

The future worth of 20 quarterly lease payments of \$500 at an interest rate of 8% is nearest to:

- a. \$8,176
- b. \$8,339
- c. \$12,149
- d. \$12,392

**Solution**

$$\begin{aligned}FW &= [500 + 500(P/A, 2\%, 19)](F/P, 2\%, 20) \\ &= \$12,391.75\end{aligned}$$

Alternate solution:

$$\begin{aligned}FW &= [500(F/P, 2\%, 1)](F/A, 2\% 20) \\ &= \$12,391.47\end{aligned}$$

The answer is d.

**8-9**

A new automobile offers free maintenance during the first year of ownership. The maintenance costs the second year are estimated to be \$100 and to increase by \$100 each year thereafter. Assume you are planning on owning the automobile 5 years and that your cost of money is 8%. The future worth of the maintenance costs is nearest to:

- a. \$683
- b. \$737
- c. \$1,083
- d. \$1,324

**Solution**

$$\begin{aligned}FW &= 100(P/G, 8\%, 5)(F/P, 8\%, 5) \\ &= \$1,083\end{aligned}$$

The answer is c.

**8-10**

Zill, Anderson, and Pope (ZAP) Bug Killers Inc. recently purchased new electrical shock equipment guaranteed to kill any flying insect. The equipment cost \$16,250 and has a useful life of 4 years. Each year the equipment will result in income of \$5,500. The costs incurred to operate the machine are estimated to be \$500 the first year and increase by \$250 year thereafter. When the equipment is disposed of it is expected to have a value of \$800. If ZAP's MARR is 8%, what is the net future worth of the equipment? Was the purchase a wise investment?

**Solution**

$$NFW = -16,250(F/P, 8\%, 4) + [5,000 - 250(A/G, 8\%, 4)](F/A, 8\%, 4) + 800$$

= -\$351.61 → Not a wise investment.

**8-11**

Tuff Nuts Inc. must buy a new nut cracking machine. The industrial engineer has collected the following information concerning the apparent best alternative. Calculate the future worth of the alternative if the MARR = 6%

First Cost	\$250,000
Annual Benefits	73,000 the first year and decreasing by \$1,200 each year thereafter
Annual O & M Costs	28,000 the first year and increasing by \$1,600 each year thereafter
Salvage Value	42,000
Useful Life	6 years

**Solution**

$$\begin{aligned} \text{NFW} &= -250,000(F/P, 6\%, 6) + [45,000 - 2,800(A/G, 6\%, 6)](F/A, 6\%, 6) + 42,000 \\ &= -\$44,380 \end{aligned}$$

## PAYBACK PERIOD

**8-12**

For calculating payback period, when is the following formula valid?

$$\text{Payback period} = \frac{\text{First Cost}}{\text{Annual Benefits}}$$

**Solution**

- Valid when:
- a) There is a single cost occurring at time zero (first cost).
  - b) Annual Benefits = Net annual benefits after subtracting any annual costs
  - c) Net Annual Benefits are uniform

**8-13**

Is the following statement True or False?

If two investors are considering the same project, the payback period will be longer for the investor with the higher minimum attractive rate of return (MARR).

**Solution**

Since payback period is generally the time to recover the investment, and ignores the MARR, it will be the same for both investors. The statement is False.

**8-14**

What is the payback period for a project with the following characteristics, if the minimum

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attractive rate of return (MARR) is 10%?

First Cost	\$20,000
Annual Benefits	8,000
Annual Maintenance	2,000 in year 1, then increasing by \$500 per year
Salvage Value	2,000
Useful Life	10 years

**Solution**

Payback occurs when the sum of net annual benefits is equal to the first cost. Time value of money is ignored.

<u>Year</u>	<u>Benefits</u>	-	<u>Costs</u>	=	<u>Net Benefits</u>	<u>Total Net Benefits</u>
1	8,000	-	2,000	=	6,000	6,000
2	8,000	-	2,500	=	5,500	11,500
3	8,000	-	3,000	=	5,000	16,500
4	8,000	-	3,500	=	4,500	21,000 > 20,000

Payback period = 4 years (actually a little less)

**8-15**

Determine the payback period (to the nearest year) for the following project if the MARR is 10%.

First Cost	\$10,000
Annual Maintenance	500 in year 1, increasing by \$200 per year
Annual Income	3,000
Salvage Value	4,000
Useful Life	10 years

**Solution**

<u>Year</u>	<u>Net Income</u>	<u>Sum</u>
1	2,500	2,500
2	2,300	4,800
3	2,100	6,900
4	1,900	8,800
5	1,700	10,500 > 10,000

Payback period = 5 years

**8-16**

Determine the payback period (to the nearest year) for the following project:

Investment Cost	\$22,000
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Annual Maintenance Costs	1,000
Annual Benefits	6,000
Overhaul Costs	7,000 every 4 years
Salvage Value	2,500
Useful Life	12 years
MARR	10%

**Solution**

<u>Year</u>	<u>Σ Costs</u>	<u>Σ Benefits</u>	
0	22,000	--	
1	23,000	6,000	
2	24,000	12,000	
3	25,000	18,000	
4	33,000	24,000	
5	34,000	30,000	
6	35,000	36,000	← Payback

Payback period = 6 years

**8-17**

Fish-or-Cut-Bait excursion boats has just purchased a new 22 passenger skimmer for use over the next 10 years. The cost of the boat was \$80,000. The income associated with the boat is expected to be \$15,000 each year and the costs are estimated to be \$2,000 the first year and increase by \$500 per year each year thereafter. The salvage value of the boat is estimated to be \$5,000. If the MARR is 5%, what is the payback for the boat?

**Solution**

<u>Year</u>	<u>Income</u>	<u>Costs</u>	<u>Net Income</u>	<u>Sum</u>
1	15,000	2,000	13,000	13,000
2	15,000	2,500	12,500	25,500
3	15,000	3,000	12,000	37,500
4	15,000	3,500	11,500	49,000
5	15,000	4,000	11,000	60,000
6	15,000	4,500	10,500	70,500
7	15,000	5,000	10,000	80,500 > 80,000

Payback period = 5 years

**8-18**

A new soap press purchased by the Rub-a-Dub-Dub Soap Company cost \$70,000. Determine the discounted payback in months if the press can produce 120 gross bars each month and each bar is sold for \$.96 and cost \$.42 to produce. Use an interest rate of 12%. (1 gross = 144 items)

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Month	Cash Flow	Discounted CF*	PB <sub>t</sub>	
0	-70,000.00		-70,000	
1	9,331.20	9,238.82	-60,761	
2	"	9,147.38	-51,614	
3	"	9,056.86	-42,557	
4	"	8,967.28	-33,590	
5	"	8,878.64	-24,711	
6	"	8,789.99	-15,921	
7	"	8,703.21	-7,218	
8	"	8,617.36	1,339	→Discounted payback period equal 8 months

\*Discounted CF= CF(P/A, 1%, i)

**BREAKEVEN****8-19**

A road can be paved with either asphalt or concrete. Concrete costs \$15,000/km and lasts 20 years. What is the maximum that should be spent on asphalt if it only lasts 10 years? Annual maintenance costs for both pavements are \$500/km. MARR = 12%.

**Solution**

Since maintenance is the same for both, it doesn't affect the answer. However, there is nothing wrong with including it.

$$15,000(A/P, 12\%, 20) = P_{\text{ASPHALT}}(A/P, 12\%, 10)$$

$$15,000(.1339) = P_{\text{ASPHALT}}(.1770)$$

$$P_{\text{ASPHALT}} = \$11,347$$

**8-20**

A machine that produces a certain piece must be turned off by the operator after each piece is completed. The machine "coasts" for 15 seconds after it is turned off, thus preventing the operator from removing the piece quickly before producing the next piece. An engineer has suggested installing a brake that would reduce the coasting time to 3 seconds.

The machine produces 50,000 pieces a year. The time to produce one piece is 1 minute 45 seconds, excluding coasting time. The operator earns \$8.00 an hour and other direct costs for operating the machine are \$4.00 an hour. The brake will require servicing every 500 hours of operation. It will take the operator 30 minutes to perform the necessary maintenance and will require \$44.00 in parts and material. The brake is expected to last 7500 hours of operation (with proper maintenance) and will have no salvage value.

How much could be spent for the brake if the minimum attractive rate of return is 10%

compounded annually?

**Solution**

$$\text{Annual cost w/o the brake} = 50,000 \left( \frac{2}{60} \right) (12) = \$20,000$$

$$\text{Annual cost w/ the brake} = 50,000 \left( \frac{1.8}{60} \right) (12) = \$18,000$$

$$\text{Maintenance} = \left[ \frac{\left( 50,000 \left( \frac{1.8}{60} \right) \right)}{500} \right] (5(12 + 44)) = \$150$$

$$\text{Brake will last: } \frac{7,500}{\left( \frac{5,000(1.8)}{60} \right)} = 5 \text{ years}$$

$$\text{Max. amount} = (20,000 - 18,150)(P/A, 10\%, 5) = \$7,013.35$$

**8-21**

A proposed building may be roofed in either galvanized steel sheet or composition roofing. The composition roof costs \$20,000 and must be replaced every 5 years at the same cost. The steel roof costs \$28,000 but the useful life is unknown. Neither roof has any salvage value and no maintenance is needed. If the minimum attractive rate of return (MARR) equals 15%, what is the minimum life that the steel roof must have to make it the better alternative? (Report to the nearest whole year; don't bother interpolating.)

**Solution**

$$EAC_S = EAC_C$$

$$28,000(A/P, 15\%, n) = 20,000(A/P, 15\%, 5)$$

$$28,000(A/P, 15\%, n) = 5,966$$

$$(A/P, 15\%, n) = .2131$$

$$(A/P, 15\%, 8) = .2229$$

$$(A/P, 15\%, 9) = .2096 \quad \rightarrow \quad n = 9$$

**8-22**

What is the breakeven capital cost for Project B compared to Project A if interest equals 10%?

<u>Year</u>	<u>A</u>	<u>B</u>
0	-1,000	?
1 - 5	+300/year	+200/year

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### **Solution**

$$\begin{aligned}\text{NPW of A} &= -1,000 + 300(P/A, 10\%, 5) \\ &= 137.3\end{aligned}$$

$$\begin{aligned}\text{NPW of B} &= \text{NPW of A} \\ 137.3 &= P_B + 200(P/A, 10, 5) \\ P_B &= -\$620.90\end{aligned}$$

### **8-23**

What is the smallest acceptable annual income from a project that has a \$70,000 investment cost and a \$70,000 salvage value if the life is 15 years and the minimum attractive rate of return (MARR) is 20%?

### **Solution**

$$\begin{aligned}\text{Income} &= 70,000(A/P, 20\%, 15) - 70,000(A/F, 20\%, 15) \\ &= 70,000[(A/P, 20\%, 15) - (A/F, 20\%, 15)] \\ &= \$14,000\end{aligned}$$

### **8-24**

A car rental agency has a contract with a garage to have them do major repairs (specified in the contract) for \$450/car every six months. The car rental agency estimates that for \$150,000, amortized at 8% interest for 20 years, and a salvage value of \$60,000, they could have their own facility. They estimate that they could take care of their own car repairs in this facility at a cost of \$200/car every six months. Ignoring taxes and other economic factors, what is the minimum number of cars needed to make the change feasible?

### **Solution**

$$\begin{aligned}\text{Let } N &= \text{number of autos needed} \\ 450N &= (150,000 - 60,000)(A/P, 4\%, 40) + 60,000(.04) + 200N \\ 450N &= 90,000 \times .0505 + 2,400 + 200N \\ 250N &= 6,945 \\ N &= 27.78 \text{ or } 28 \text{ autos needed}\end{aligned}$$

### **8-25**

Assume you need to buy some new automobile tires and you are considering purchasing either the "Econo-Ride", which costs \$33.95 per tire, or the "Road King", which costs \$65.50. Both tires are alike except that the "Road King" is more durable and will last longer. Regardless of which tire is purchased, balancing and installation costs are \$1.50 per tire. The salesman says the "Econo-Ride" will last 20,000 miles. Assume a minimum attractive rate of return (MARR) of 6% and that you drive 10,000 miles per year.

- How many miles would the "Road King" have to last to make you indifferent in your choice?
- The salesman says the "Road King" will be on sale next week. If he also says the tire will last 30,000 miles, what would the sale price have to be to make you indifferent in your choice?

**Solution**

- a.  $4(1.5 + 33.95)(A/P, 6\%, \frac{20,000}{10,000}) = 4(1.5 + 65.50)(A/P, 6\%, N)$   
 $(A/P, 6\%, N) = .28859$   
 From tables  $N = 4$ , or 40,000 miles
- b.  $4(1.5 + 33.95)(A/P, 6\%, \frac{20,000}{10,000}) = 4(1.5 + P)(A/P, 6\%, \frac{30,000}{10,000})$   
 $P = \$50.18$

**8-26**

A machine, costing \$2,000 to buy and \$300 per year to operate, will save labor expenses of \$650 per year for 8 years. If the interest rate is 10%, what is the minimum salvage value (after 8 years) at which the machine is worth purchasing?

**Solution**

$$\begin{aligned} NPW &= -2,000 + 350(P/A, 10\%, 8) + S(P/F, 10\%, 8) \\ 0 &= -132.75 + .4665S \\ S &= \$284.57 \end{aligned}$$

**8-27**

A soft drink company has researched the possibility of marketing a new low-calorie beverage, in a study region. The expected profits depend largely on the sales volume, and there is some uncertainty as to the precision of the sales-forecast figures. The estimated investment is \$173,000 while the anticipated profits are \$49,500 per year for the next 6 years. If the company's MARR = 15%, is the decision to invest sensitive to the uncertainty of the sales forecast, if it is estimated that in the worst case the profits will be reduced to \$40,000 per year? What is the minimum volume of sales for the project to breakeven, if there is a profit of \$6.70 per unit volume?

**Solution**

- a. For an annual profit of \$49,500  
 $NPW = 49,500 (P/A, 15\%, 6) - 173,000$   
 $= +14,308$  (attractive)
- b. For an annual profit of \$40,000  
 $NPW = 40,000 (P/A, 15\%, 6) - 173,000$   
 $= -21,640$  (not attractive)

Therefore the decision is sensitive to the expected variations in sales or profits.

The breakeven:  $NPW = 0$

$NPW = 0 = X(P/A, 15\%, 6) - 173,000$  where  $X = \text{min } \$ \text{ profit}$

$$X = \frac{173,000}{(P/A, 15\%, n)} = \$45,718.80$$

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$$\text{in volume units} = \frac{\$45,718.80}{\$6.70/\text{unit}} = 6,824 \text{ volume units}$$

### 8-28

The PARC Company can purchase gizmos to be used in building whatsits for \$90 each. PARC can manufacture their own gizmos for \$7,000 per year overhead cost plus \$25 direct cost for each gizmo, provided they purchase a gizmo maker for \$100,000. PARC expects to make whatsits using gizmos for 10 years. The gizmo maker should have a salvage value of \$20,000 after 10 years. PARC uses 12% as its minimum attractive risk rate. At what annual production rate should PARC make their own gizmos?

### Solution

Equivalent Cost Solution:

$$EAC_{\text{BUY}} = \$90N \text{ where } N = \text{annual quantity}$$

$$EAC_{\text{MAKE}} = 100,000(A/P, 12\%, 10) + 7,000 + 25N - 20,000(A/F, 12\%, 10) \\ = 23,560 + 25N$$

For breakeven:

$$EAC_{\text{BUY}} = EAC_{\text{MAKE}}$$

$$90N = 23,560 + 25N$$

$$N = 362.5$$

This indicates they should be bought at 362/year or less and made at 363/year or more.

## PRESENT WORTH INDEX

### 8-29

Taylor Technologies is considering buying new circuit analyzing software. The software license sells for \$12,500 for a five-year usage period. The software speeds up the circuit design process and is estimated to yield savings of \$6,000 per year. Upgrades and updates will cost the company \$2,000 annually. Taylor uses the present worth index to make all purchasing decisions. The company's MARR is 7%. Should Taylor Tech. purchase the software?

### Solution

$$PW_{\text{CF}} = 6,000(P/A, 7\%, 5) - 2,000(P/A, 7\%, 5) = \$16,400$$

$$PW_{\text{FC}} = \$12,500$$

$$PWI = \frac{PW_{\text{CF}}}{PW_{\text{FC}}} = \frac{16,400}{12,500} = 1.31$$

Taylor Tech. should buy the software

**8-30**

The Bolivar Nut Farm is considering the purchase of a new walnut shelling machine to replace human nut crushers. The farm currently pays four human nutcrackers \$15,000 per year. The automated sheller has a first cost of \$200,000. Annual end of the year maintenance for the machine will be \$10,000 the first year and increase by \$1,000 per year. The machine will cost \$1,000 per year for operating cost. The machine will have a salvage value of \$10,000 at the end of its five year useful life. If the cost of money for the Bolivar Nut farm is 6%, determine the present worth index. Based on the PWI should the machine be purchased?

**Solution**

$$PWI = \frac{PW_{\text{CASH FLOWS}}}{PW_{\text{FIRST COST}}}$$

$$\begin{aligned} PW_{CF} &= (4)(15,000)(P/A, 6\%, 5) - 1000(P/A, 6\%, 5) \\ &\quad - [10,000(P/A, 6\%, 4) + 1,000(P/G, 6\%, 4)] + 10,000(P/F, 6\%, 5) \\ &= \$216,386 \end{aligned}$$

$$PW_{FC} = \$200,000$$

$$PWI = \frac{216,386}{200,000} = 1.08$$

$PWI > 1$       Purchase the machine

**8-31**

A construction co. wants to purchase a new skid steer. The first cost is \$30,000. The yearly savings is \$10,800. Operating and maintenance is \$3,600 the first year and increases \$500 per year. The salvage value is expected to be \$1,000. The estimated life is 10 years. The company's MARR is 7%. Find the present worth index.

**Solution**

$$\begin{aligned} PW_{CF} &= 10,800(P/A, 7\%, 10) - [3,600(P/A, 7\%, 10) + 500(P/G, 7\%, 10)] - 1,000(P/F, 7\%, 10) \\ &= \$37,223 \end{aligned}$$

$$PW_{FC} = \$30,000$$

$$PWI = \frac{PW_{CF}}{PW_{FC}} = \frac{37,223}{30,000} = 1.24$$