

Chapter 7

Internal Rate of Return

7-1

Andrew T. invested \$15,000 in a high yield account. At the end of 30 years he closed the account and received \$539,250. Compute the effective interest rate he received on the account.

Solution

Recall that $F = P(1 + i)^n$

$$539,250 = 15,000(1 + i)^{30} \Rightarrow 539,250/15,000 = (1 + i)^{30}$$

$$35.95 = (1 + i)^{30}$$

$$\sqrt[30]{35.95} = 1 + i$$

$$1.1268 = 1 + i$$

$$1268 = i$$

$$i = 12.68\%$$

7-2

The heat loss through the exterior walls of a processing plant is estimated to cost the owner \$3,000 next year. A salesman from Superfiber, Inc. claims he can reduce the heat loss by 80% with the installation of \$15,000 of Superfiber now. If the cost of heat loss rises by \$200 per year, after next year (gradient), and the owner plans to keep the building ten more years, what is his rate of return, neglecting depreciation and taxes?

Solution

NPW = 0 at the rate of return

Try 12%

$$\begin{aligned} \text{NPW} &= -15,000 + .8(3,000)(P/A, 12\%, 10) + .8(200)(P/G, 12\%, 10) \\ &= \$1,800.64 \end{aligned}$$

Try 15%

$$\text{NPW} = -\$237.76$$

By interpolation $i = 14.7\%$

7-3

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Does the following project have a positive or negative rate of return? Show how this is known to be true.

Investment Cost	\$2,500
Net Benefits	\$300 in Year 1, increasing by \$200 per year
Salvage	\$50
Useful Life	4 years

Solution

<u>Year</u>	<u>Benefits</u>	
1	300	Total Benefits obtained are less than the investment, so the "return" on the investment is negative.
2	500	
3	700	
4	900	
4	<u>50</u>	
Total = \$2,450 < Cost		

7-4

At what interest rate would \$1,000 at the end of 2000 be equivalent to \$2,000 at the end of 2007?

Solution

$$(1 + i)^7 = 2; \quad i = (2)^{1/7} - 1 = 0.1041 \text{ or } 10.41\%$$

7-5

A painting, purchased one month ago for \$1,000, has just been sold for \$1,700. What nominal annual rate of return did the owner receive on her investment?

Solution

$$i = 7,000/1,000 = 70\% \quad r = 70 \times 12 = 840\%$$

7-6

Find the rate of return for a \$10,000 investment that will pay \$1,000/year for 20 years.

Solution

$$10,000 = 1,000(P/A, i\%, 20) \\ (P/A, i\%, 20) = 10$$

$$\text{From interest tables: } 7\% < i < 8\% \quad \therefore \text{interpolate } i = 7.77\%$$

7-7

A young engineer has a mortgage loan at a 12% interest rate, which she got some time ago, for a total of \$52,000. She has to pay 240 more monthly payments of \$534.88 each. As interest rates are going down, she inquires about the conditions under which she could refinance the loan. If the bank charges a new loan fee of 2% of the amount to be financed, and if the bank and the engineer

agree on paying this fee by borrowing the additional 2% under the same terms as the new loan, what percentage rate would make the new loan attractive, if the conditions require her to repay it in 120 payments?

Solution

The amount to be refinanced:

$$i = 12/12 = 1\%$$

a) PW of 120 monthly payments left = $534.88(P/A, 1\%, 240)$
 $= \$48,577.27$

b) New loan fee (2%) = $48,577.27(.02) = \$971.55$

⇒ Total amount to refinance = $48,577.27 + 971.55 = \$49,548.82$

The new monthly payments are: $A_{NEW} = 49,548.82(A/P, i, 120)$

The current payments are: $A_{OLD} = 534.88$

We want $A_{NEW} < A_{OLD}$

Substituting ⇒ $49,548.82(A/P, i, 120) < 534.88$

$$(A/P, i, 120) < 534.88/49,548.82 = 0.0108$$

for $i = 1/4\%$ $(A/P, 1/4\%, 120) = 0.00966$

for $i = 1/2\%$ $(A/P, 1/2\%, 120) = 0.0111$

$1/4\% < i < 1/2\%$ ∴ interpolate

$$i = .4479\%$$

This corresponds to a nominal annual percentage rate of $12 \times 0.4479 = 5.375\%$
 Therefore, she has to wait until interest rates are less than 5.375%.

7-8

Your company has been presented with an opportunity to invest in a project. The facts on the project are presented below:

Investment Required	\$60,000,000
Annual Gross Income	14,000,000
Annual Operating Costs	5,500,000
Salvage Value after 10 Years	0

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The project is expected to operate as shown for ten years. If your management expects to make 10% on its investments before taxes, would you recommend this project?

Solution

$$\begin{aligned}\text{Net income} &= 14,000,000 - 5,500,000 \\ &= \$8,500,000\end{aligned}$$

NPW = 0 at the rate of return

$$\begin{aligned}0 &= -60,000,000 + 8,500,000(P/A, i, 10) \\ (P/A, i, 10) &= 60/8.5 \\ &= 7.0588\end{aligned}$$

$$\begin{aligned}\text{@6\%} & \quad P/A = 7.360 \\ \text{@7\%} & \quad P/A = 7.024\end{aligned}$$

6% < i < 7% ∴ interpolate

$$i = 6.9\%$$

IRR < 10 % ∴ Do Not Recommend Project

7-9

Consider the following investment in a piece of land.

Purchase price	\$10,000
Annual maintenance:	100
Expected sale price after 5 years:	\$20,000

Determine:

- A trial value for i
- The rate of return (to 1/100 percent)
- What is the lowest sale price the investor should accept if she wishes to earn a return of 10% after keeping the land for 10 years?

Solution

$$\begin{aligned}\text{(a)} \quad (F/P, i \%, 5) &= 20,000/10,000 \\ &= 2\end{aligned}$$

Searching interest tables where n = 5

$$i = 15 \%$$

$$\text{(b)} \quad \text{NPW} = -10,000 - 100(P/A, i \%, 5) + 20,000(P/F, i \%, 5) = 0$$

$$\text{Try } i = 15\%: = -391.2$$

$$\text{Try } i = 12\%: = +987.5$$

$15\% < i < 12\%$ \therefore interpolate

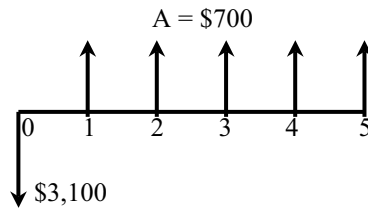
$$i = 14.15\%$$

(c) $NFW = 0 = -10,000(F/P, 10\%, 10) - 100(F/A, 10\%, 10) + \text{Sale Price}$

$$\text{Sale Price} = \$27,534$$

7-10

Calculate the rate of return of the following cash flow with accuracy to the nearest 1/10 percent.



Solution

NPW = 0 at the rate of return

$$0 = -3,100 + 700(P/A, i, 5)$$

$$(P/A, i, 5) = 4.4286$$

$$(P/A, 4\%, 5) = 4.452$$

$$(P/A, 4\frac{1}{2}\%, 5) = 4.390$$

$4\% < i < 4\frac{1}{2}\%$ \therefore interpolate

$$i = 4.2\%$$

7-11

An investment that cost \$1,000 is sold five years later for \$1,261. What is the nominal rate of return on the investment if interest is compounded annually?

Solution

$$F = P(F/P, i\%, 5)$$

$$1,261 = 1,000(F/P, i\%, 5)$$

$$(F/P, i\%, 5) = 1,261/1,000$$

$$= 1.2610$$

$$(F/P, 4\frac{1}{2}\%, 5) = 1.246$$

$$(F/P, 5\%, 5) = 1.276$$

$4\frac{1}{2}\% < i < 5\%$ \therefore interpolate

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$$i = 4.75\%$$

7-12

Lexie C. made an initial investment of \$5,000 in a trading account with a stock brokerage house. After a period of 17 months the value of the account had increased to \$6,400. What is the nominal annual interest rate earned on the initial investment if it is assumed there were no additions or withdrawals from the account?

Solution

$$\begin{aligned} F &= P(F/P, i, 17) \\ F/P &= 6,400/5,000 \\ &= 1.28 \end{aligned}$$

$$\begin{aligned} (1 + i)^{17} &= 1.28 \\ 1 + i &= (1.28)^{1/17} \\ i &= 0.0146 \end{aligned}$$

$$\text{Annual interest rate} = 1.46 \times 12 = 17.52\%$$

7-13

Whiplash Airbags has been presented an investment opportunity that is summarized below.

Year	0	1	2	3	4	5	6	7	8
Cash Flow (1000's)	\$(440)	20	40	60	80	100	120	140	160

Determine the IRR for the proposed investment.

Solution

NPW = 0 at the rate of return

Try 8%

$$\begin{aligned} \text{NPW} &= -440,000 + 20,000(P/A, 8\%, 8) + 20,000(P/G, 8\%, 8) \\ &= \$31,060 \end{aligned}$$

Try 10%

$$\text{NPW} = -\$12,720$$

By interpolation $i = 9.42\%$

7-14

You have a choice of \$2,000 now, or \$250 now with \$80 a month for two years. What interest rate will make these choices comparable?

Solution

$$2,000 = 250 + 80(P/A, i, 24)$$

$$P/A = 21.875$$

@ 1% P/A = 21.243
 @ ¾% P/A = 21.889

¾% < i < 1% ∴ interpolate

i = .7554% per month or 9.07% per year

7-15

Joe’s Billiards Inc. stock can be purchased for \$14.26 per share. If dividends are paid each quarter at a rate of \$0.16 per share, determine the effective *i* if after 4 years the stock is sold for \$21.36 per share.

Solution

¼ Year		3%	3½
0	Cost	-14.26	-14.26
1-16	Dividends .16(P/A, i%, 16)	+2.01	+1.93
16	Sale 21.36(P/F, i%, 16)	<u>+13.31</u>	<u>+12.32</u>
	NPW =	\$1.06	-\$0.1

i = 3½ per quarter

$$i_{\text{eff}} = (1 + .035)^4 - 1$$

$$= 14.75\%$$

7-16

A 9.25% coupon bond issued by Gurley Gears LLC is purchased January 1, 2011 and matures December 31, 2019. The purchase price is \$1,079 and interest is paid semi-annually. If the face value of the bond is \$1,000, determine the effective internal rate of return.

Solution

n = (2)(9) = 18 ½-year periods

½ Year		4%
0	First Cost	-1,079.00
1-18	Interest 46.25(P/A, i%, 18)	+585.48
18	Maturity 1000(P/F, i%, 18)	<u>+493.60</u>
	NPW =	\$.08

$$IRR = (1 + .04)^2 - 1$$

$$= 8.16\%$$

7-17

Sain and Lewis Investment Management (SLIM) Inc., is considering the purchase of a number of bonds to be issued by Southeast Airlines. The bonds have a face value of \$10,000 and a face rate

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of 7.5% payable annually. The bonds will mature ten years after they are issued. The issue price is expected to be \$8,750. Determine the yield to maturity (IRR) for the bonds. If SLIM Inc. requires at least a 10% return on all investments should they invest in the bonds?

Solution

Year		10%	9%
0	First Cost	-8,750.00	-8,750.00
1-10	Interest 750(P/A, i%, 10)	+4,608.75	+4,813.50
10	Maturity 10,000(P/F, i% n)	<u>+3,855.00</u>	<u>+4,224.00</u>
	NPW =	-\$286.25	\$287.50

9% < IRR < 10% ∴ interpolate

i = 9.5% ⇒ Do not invest.

7-18

A bond with a face value of \$1,500 can be purchased for \$800. The bond will mature five years from now and the bond dividend rate is 12%. If dividends are paid every three months, what effective interest rate would an investor receive if she purchases the bond?

Solution

NPW = 0 at IRR

Try 7%

$$\begin{aligned} \text{NPW} &= -800 + 45(\text{P/A}, 7\%, 20) + 1,500(\text{P/F}, 7\%, 20) \\ &= \$64.33 \end{aligned}$$

Try 8%

$$\text{NPW} = -\$36.44$$

7% < i < 8% ∴ interpolate

i = 7.64% per quarter

$$\text{Effective interest rate} = (1 + 0.07638)^4 - 1 = 0.3423 = 34.23\%$$

7-19

Milton Hotels Inc. stock is currently selling for \$20.75. A dividend of 35¢ per share is paid semi-annually. The stock has increased in price by 5% annually for the last several years. Assuming the stock continues to increase in price at the same rate and does not increase the dividend amount, what is the IRR if an investor sells the stock after four years?

- 4.00%
- 5.00%
- 8.00%

d. 8.25%

Solution

$$\text{Projected resale value} = 1.05^4(20.75) = \$25.22$$

NPW = 0 at IRR

Try 4%

$$\begin{aligned} \text{NPW} &= -20.75 + .35(P/A, 4\%, 8) + 25.22(P/F, 4\%, 8) \\ &= \$.04 \end{aligned}$$

Try 4½%

$$\text{NPW} = -\$.70$$

4% < i < 4½% ∴ interpolate

i = 4.03% per period

$$\text{Effective interest rate} = (1 + 0.00403)^2 - 1 = 8.22\%$$

The answer is d.

7-20

An investment of \$350,000 is made, followed by income of \$200,000 each year for three years. There is no residual value at the end of three years. The IRR of the investment is nearest to:

- a. 15.3%
- b. 32.7%
- c. 41.7%
- d. 57.1%

Solution

NPW = 0 at IRR

$$\begin{aligned} 0 &= -350,000 + 200,000(P/A, i\%, 3) \\ (P/A, i\%, 3) &= 350,000/200,000 \\ &= 1.75 \end{aligned}$$

@ 30% P/A = 1.816

@ 35% P/A = 1.696

The answer is b.

7-21

Tri-state Tire is considering the purchase of new inflation equipment for its Martin operation. Cash flows associated with the new equipment are presented below. Determine the IRR.

<u>YR</u>	<u>Cash Flow</u>
0	\$(2,000)
1	1,000

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2	750
3	500
4	250
5	0
6	-250

Solution

NPW = 0 at IRR

Try 7%

$$\begin{aligned} \text{NPW} &= -2,000 + 1,000(P/A, 7\%, 6) - 250(P/G, 7\%, 6) \\ &= \$20.51 \end{aligned}$$

Try 8%

$$\text{NPW} = -\$7.35$$

7% < i < 8% ∴ interpolate

i = 7.73%