Chapter 6

Equivalent Annual Worth

6-1

Deere Construction just purchased a new track hoe attachment costing $12,500. The CFO, John, expects the implement will be used for five years when it is estimated to have a salvage value of $4,000. Maintenance costs are estimated to be $0 the first year and will increase by $100 each year thereafter. If a 12% interest rate is used, what is the equivalent uniform annual cost of the implement?

a. $2,925
b. $2,975
c. $3,015
d. $3,115

Solution

EUAC = 12,500(A/P, 12%, 5) - 4,000(A/F, 12%, 5) + 100(A/G, 12%, 5)
= $3,015.40

The answer is c.

6-2

The survey firm of Myers, Anderson, and Pope (MAP) LLP is considering the purchase of a piece of new GPS equipment. Data concerning the alternative under consideration are presented below.

First Cost	$28,000
Annual Income	7,000
Annual Costs	2,500
Recalibration at end of Year 4	4,000
Salvage Value	2,800

If the equipment has a life of eight years and MAP's minimum attractive rate of return (MARR) is 5%, what is the annual worth of the equipment?

Solution

EUAC = 28,000(A/P, 5%, 8) - 4,500 - 4,000(P/F, 5%, 4)(A/P, 5%, 8) - 2,800(A/F, 5%, 8)
= -$47.63

6-3

Ronald McDonald decides to install a fuel storage system for his farm that will save him an estimated 6.5 cents/gallon on his fuel cost. He uses an estimated 20,000 gallons/year on his farm. Initial cost of the system is $10,000 and the annual maintenance the first year is $25 and increases by $25 each year thereafter. After a period of 10 years the estimated salvage is $3,000. If money is worth 12%, is it a wise investment?

Solution

$$EUAC = 10,000(A/P, 12\%, 10) + 25 + 25(A/G, 12\%, 10)$$
$$= \$1,884.63$$

$$EUAB = 20,000(.065) + 3,000(A/F, 12\%, 10) = \$1,471.00$$

$$EUAW = -\$413.63 \therefore \text{ not a wise investment}$$

6-4

The incomes for a business for five years are as follows: $8,250, $12,600, $9,750, $11,400, and $14,500. If the value of money is 12%, what is the equivalent uniform annual benefit for the five-year period?

Solution

$$PW = 8,250(P/F, 12\%, 1) + 12,600(P/F, 12\%, 2) + 9,750(P/F, 12\%, 3)$$
$$+ 11,400(P/F, 12\%, 4) + 14,500(P/F, 12\%, 5)$$
$$= \$39,823$$

$$EUAB = 39,823(A/P, 12\%, 5) = \$11,047$$

6-5

At an interest rate of 10% per year, the perpetual equivalent annual cost of $70,000 now, $100,000 at the end of year six, and $10,000 per year from the end of year ten through infinity is closest to:

   a. $16,510
   b. $24,200
   c. $31,500
   d. $37,630

Solution

$$P = 70,000 + 100,000(P/F, 10\%, 6) + 10,000(P/A, 10\%, \infty)(P/F, 10\%, 10)$$
$$= \$165,110$$

$$A = 165,110(A/P, 10\%, \infty)$$
$$= \$16,511$$

The answer is a.

6-6

The state engineer estimates that the cost of a canal is $680 million. The legislative analyst estimates the equivalent annual cost of the investment for the canal to be $20.4 million. If the analyst expects the canal to last indefinitely, what interest rate is he using to compute the

equivalent annual cost (EAC)? If the canal lasts only 50 years, what interest rate will the analyst be assuming if he believes the EAC to be the same $20.4 million?

Solution

a) $A = P(A/P, i\%, n)$
      For $n = \infty$, $(A/P, i\%, \infty) = i$
         $A = P(i)$

         $i = A/P = \dfrac{20.4}{680} = .03$ or $i = 3\%$

b) $A = P (A/P, i\%, 50)$

      $(A/P, i\%, 50) = \dfrac{20.4}{680} = .03$

      Searching interest tables at $n = 50$ $i = 1.75\%$

6-7

What uniform annual payment for 12 years is equivalent to receiving all of the following:

         $ 3,000 at the end of each year for 12 years
         20,000 today
          4,000 at the end of 6 years
            800 at the end of each year forever
         10,000 at the end of 15 years

Use an 8% interest rate.

Solution

   $A_1 = \$3,000$
   $A_2 = 20,000(A/P, 8\%, 12) = \$2,654$
   $A_3 = 4,000(P/F, 8\%, 6)(A/P, 8\%, 12) = \$334.51$
   $A_4 = (800/.08)(A/P, 8\%, 12) = \$1,327$
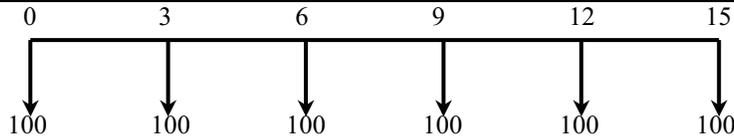 $A_5 = 10,000(P/F, 8\%, 15)(A/P, 8\%, 12) = \$418.27$

$$\sum_{i=1}^{n} Ai \; = \; 3{,}000 \; + \; 2{,}654 \; + \; 334.51 \; + \; 1{,}327 \; + \; 418.27 \; = \; \$7{,}733.78$$

6-8

For the following cash flow diagram, which equation properly calculates the uniform equivalent?

a. $A = 100(A/P, i, 3) + 100(A/F, i, 3)$
b. $A = 100(A/P, i, 15)$
c. $A = 100(A/P, i, 15) + 100(A/F, i, 3)$
d. $A = 100(A/F, i, 3) + 100(A/F, i, 15)$

```
        0       3       6       9       12      15
        |       |       |       |       |       |
        ↓       ↓       ↓       ↓       ↓       ↓
      100     100     100     100     100     100
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Solution

The correct equation is (c).

6-9
A project has a first cost of $75,000, operating and maintenance costs of $10,000 during each year of its 8 year life, and a $15,000 salvage value. What is its equivalent uniform annual cost (EUAC) if the interest rate is 12%?

Solution

EUAC = 75,000(A/P, 12%, 8) + 10,000 - 15,000(A/F, 12%, 8) = $23,878.00

6-10
A recent engineering graduate makes a donation of $20,000 now and will pay $375.00 per month for 10 years to endow a scholarship. If interest is 9%, what annual amount can be awarded? Assume the first scholarship will be bestowed at the end of the first year after full funding.

Solution

P = 20,000 + 375.00(P/A, ¾%, 120)
  = 49,603.25

A = Pi

  = 49,603.25(.09)
  = $4464.29 scholarship

6-11
A rich folk singer has donated $500,000 to endow a university professorial chair in Bohemian Studies. If the money is invested at 8.5%, how much can be withdrawn each year, ad infinitum, to pay the Professor of B.S.?
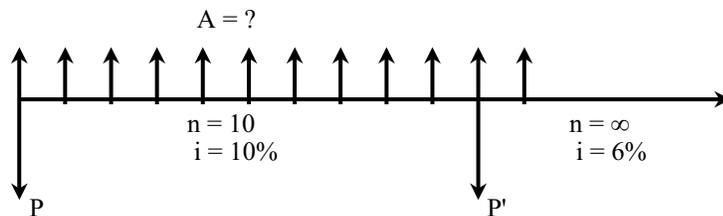
Solution

A = 500,000(A/P, 8.5%, ∞) = 500,000(.085) = $42,500

6-12
A foundation supports an annual seminar on campus by using the earnings of a $50,000 gift. It is felt that 10% interest will be realized for 10 years, but that plans should be made to anticipate an interest rate of 6% after that time. What uniform annual payment may be established from the beginning, to fund the seminar at the same level into infinity?

Solution

A = ?

n = 10
i = 10%

n = ∞
i = 6%

P

P'

Assume first seminar occurs at time of deposit.

$$P' = A/i = A/.06$$

$$P = A + A(P/A, 10\%, 10) + P'(P/F, 10\%, 10)$$
$$50,000 = A + 6.145A + (A/.06) \text{ x } .3855$$
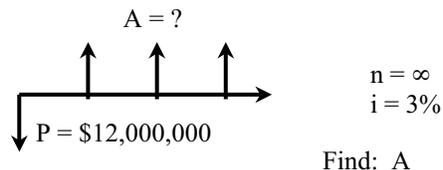$$13.57A = 50,000$$

$$A = \$3,684.60$$

6-13

Given:

A = ?

n = ∞
i = 3%

P = \$12,000,000

Find: A

Solution

$$A = Pi$$
$$= 12,000,000(0.03)$$
$$= \$360,000$$

6-14

A project requires an initial investment of \$10,000 and returns benefits of \$6,000 at the end of every 5th year thereafter. If the minimum attractive rate of return (MARR) is 10%, the equivalent uniform annual worth is closet to

a.   -\$17.20
b.   -\$1,600
c.   -\$5,000
d.   -\$8,410

Solution

Year	0	5	10	15	20	25....	... ∞
Cash Flow (\$)	-10,000	6,000	6,000	6,000	6,000	6,000	6,000

EUAW = 6,000(A/F, 10%, 5) - 10,000(A/P, 10%, ∞)
= - $17.20

The answer is a.

6-15

The first cost of a fairly large flood control dam is expected to be $5 million. The maintenance cost will be $60,000 per year, and a $100,000 outlay will be required every 5 years. At 10%, the EUAC of the dam project is closest to:

   a. $576,380
   b. $591,580
   c. $630,150
   d. $691,460

Solution

EUAC = 5,000,000(A/P, 10%, ∞) + 60,000 + 100,000(A/F, 10%, 5)
= $576,380

The answer is a.

6-16

Twenty five thousand dollars is deposited in a bank trust account that pays 9% interest, compounded semiannually. Equal annual withdrawals are to be made from the account, beginning one year from now and continuing forever. Calculate the maximum amount of the equal annual withdrawal.

Solution

i = 9/2 = 4½%

A = Pi = 25,000(.045) = 1,125 per semi-annual period

W = 1,125(F/A, 4½%, 2) = $2,300.63

6-17

Assuming monthly payments, which would be the better financing plan on the same $19,000 car?

a. 6% interest on the full amount for 48 months.
b. A $2,500 rebate (discount) and 12% interest on the remaining amount for 48 months.

Solution

   a. A = 19,000(A/P, ½%, 48) = $446.50/mo.

   b. A = 16,500(A/P, 1%, 48) = $433.95/mo.

Choose plan b.

6-18

If the interest rate is 10% and compounding is semiannual, what series of equal annual transactions is equivalent to the following series of semiannual transactions? The first of the equal annual transactions is to occur at the end of the second year and the last at the end of the fourth year.

Time (yr)	0		1		2		3		4		5	5½
Period	0	1	2	3	4	5	6	7	8	9	10	11
Cash Flow	$0	600	500	400	300	200	100	300	500	700	900	1100

Solution

$i = 10/2 = 5\%$

$P = 600(P/A, 5\%, 5) - 100(P/G, 5\%, 5) + [100(P/A, 5\%, 6) + 200(P/G, 5\%, 6)](P/F, 5\%, 5)$
$= 4,046.80$

Effective $i = (1 + 0.10/2)^2 - 1 = 10.25\%$

Sum at end of Year 1: $F = 4,046.80(F/P, 10.25\%, 1) = 4,461.60$

Equal Annual Payments: $A = 4,461.60(A/P, 10.25\%, 3) = \$1,802.04$

6-19

Smith, LYons, Carson, and Kirk (SLYCK) Inc. is considering the purchase of new petroleum processing equipment. The relevant data for the alternative under consideration are presented below.

First Cost	$278,750
Annual Income	$125/barrel of processed petroleum
Annual Operating Costs	$25,500 the first year increasing $2,000 each year thereafter
Annual Property Taxes	8% of first cost
Annual Insurance	4% of first cost payable at the beginning of each year
Salvage Value	15% of first cost
Useful Life	10 years

SLYCK's minimum attractive rate of return is 4%. Determine the number of barrels/year of petroleum that must be processed in order to justify purchasing the machine.

Solution

Year		
0	First Cost 278,750(A/P, 4%, 10)	(34,369.88)
1-10	Income 125(X)	125(X)
1-10	Costs 25,500 + 2,000(A/G, 4%, 10)	(33,854.00)
1-10	Taxes .08(278,750)	(22,300.00)

| 0-9 | Insurance .04(278,750)(F/P, 4%, 1) | (11,596.00) |
| 10 | Salvage Value .15(278,750)(A/F, 4%,10) | 3,482.98 |

0 = -34,369.88 - 33,854.00 - 22,300.00 - 11,596.00 + 3,482.98 + 125(X)
X = 789.09 Barrels

6-20
A tractor costs $12,500 and will be used for five years when it is estimated to have a salvage value of $4,000. Maintenance costs are estimated to be a $100 the first year and increase by $100 each year thereafter. If i = 12%, what is the equivalent uniform annual cost (EUAC) for the tractor?
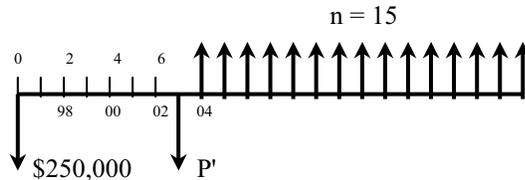
Solution

EUAC = 12,500(A/P, 12%, 5) + 100 + 100(A/G, 12%, 5) - 4,000(A/F, 12%, 5)
      = $3,115.40

6-21
If Ellen won $250,000 the last week in February, 1996 and invested it by March 1, 1996 in a "sure thing" that pays 8% interest, compounded annually, what uniform annual amount can she withdraw on the first of March for 15 years starting in 2004?

Solution

P' = 250,000(F/P, 8%, 7) = $428,500
A = 250,000(F/P, 8%, 7)(A/P, 8%, 15) = $50,048.80

6-22
A machine, with a first cost of $20,000, is expected to save $1,500 in the first year of operation and the savings should increase by $200 every year until (and including) the ninth year, thereafter the savings will decrease by $150 until (and including) the 16th year. Using equivalent uniform annual worth, is this machine economical? Assume a MARR of 10%.

Solution
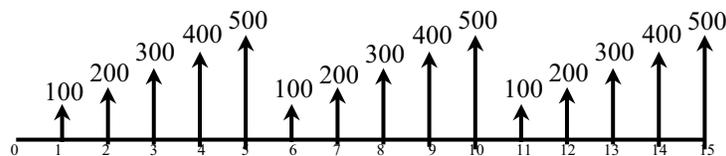
There are a number of possible solutions.
Here's one:

EUAW = -20,000(A/P, 10%, 16) + [1,500(P/A, 10%, 9) + 200(P/G, 10%, 9)](A/P, 10%, 16)
      + [2,950(P/A, 10%, 7) - 150(P/G, 10%, 7)](P/F, 10%, 9)(A/P, 10%, 16)
      = -$280.94, the machine is not economical

6-23

Calculate the equivalent uniform annual cost of the following schedule of payments.

Solution

Since payments repeat every five years, analyze for 5 years only.

A = 100 + 100(A/G, 8%, 5) = $284.60

6-24
The initial cost of a van is $12,800 and will have a salvage value of $5,500 after five years. Maintenance is estimated to be a uniform gradient amount of $120 per year (with no maintenance costs the first year), and the operation cost is estimated to be 36 cents/mile for 400 miles/month. If money is worth 12%, what is the approximate equivalent uniform annual cost (EUAC) for the van, expressed as a monthly cost?

Solution

EUAC = 12,800(A/P, 12%, 5) + 120(A/G, 12%, 5) + .36(400)(12) - (5,500)(A/F, 12%, 5)
      = 4,626/12
      = $385.50/month

6-25
Granny Gums has established a scholarship at the Martin College of Dentistry. She will make deposits into an endowment account that pays 12% per year based on the following schedule.

Year	0	1	2	3	4	5	6
Deposit Amount, $	100	90	80	70	60	50	40

If the first scholarship is to be awarded one year after the first deposit is made and will be given indefinitely, what is the scholarship amount?

a. $33.68
b. $45.68
c. $68.26
d. $92.58

Solution

First find the present worth of the gradient deposits.

P = 100 + 90(P/A, 12%, 6) - 10(P/G, 12%, 6)
  = $380.69

A = 380.69(A/P, 12%, ∞)
  = 380.69(.12)

= $45.68

The answer is b.

6-26

A proposed steel bridge has an indefinite life. The initial cost of the bridge is $3,750,000 and annual maintenance costs are estimated to be $25,000. The bridge deck will be resurfaced every 10 years for $900,000 and anticorrosion paint will be applied every 5 years for $250,000. If the interest rate is 8%, what is the EAC?

If 650,000 axles will cross the bridge each year, what is the approximate toll per axle that should be charged? Give answer to the nearest nickel.

Solution

EUAC = 3,750,000(A/P, 8%, ∞) + 900,000(A/F, 8%, 10) + 250,000(A/F, 8%, 5) + 25,000
= $429,725

Toll = 429,725/650,000
= .6611
≈ 70¢ per axle

6-27

A college is willed $100,000 to establish a permanent scholarship. If funds are invested at 6% and all funds earned are disbursed yearly, what will be the value of the scholarship in the 6th year of operation?

Solution

A = Pi = 100,000(0.06) = $6,000 for any year

6-28

The annual worth of a quarterly lease payment of $500 at 8% interest is nearest to

a. $2,061
b. $2,102
c. $2,253
d. $2,433

Solution

Recall lease payments are beginning of period.

i = 8/4 = 2%   n = 4

A = 500(F/P, 2%, 1)(F/A, 2%, 4)
= $2,102.22

Alternate solution

$A = [500 + 500(P/A, 2\%, 3)](F/P, 2\%, 4)$
  $= \$2,101.24$

The answer is b.

6-29
Two alternative investments are being considered. What is the minimum uniform annual benefit that will make Investment B preferable over Investment A? Assume interest is 10%.

Year	A	B
0	-$500	-$700
1 - 5	+150	?

Solution

$EUAW_A = EUAW_B$

$-500(A/P, 10\%, 5) + 150 = -700(A/P, 10\%, 5) + \mathbf{X}$
                $\mathbf{X} = \$202.76$

6-30
An airport expansion that is expected to be used indefinitely is currently underway at Jackson Hole Metro Airport. Land acquisition and major earthworks that will last as long as the airport is used are expected to cost $600 million. Two hundred million is budgeted for terminal construction that will last 20 years. (Assume the terminal is identically replaced every 20 years.) Runway construction will cost $150 million. The runways will also be used indefinitely, with repaving required every 10 years at a cost of $10 million. The operating and maintenance costs are estimated to be $15 million per year. What is the annual cost of the project if i = 5% and the airport will be used indefinitely?

Solution

$EUAC = 600(A/P, 5\%, \infty) + 200(A/P, 5\%, 20) + 150(A/P, 5\%, \infty) + 10(A/F, 5\%, 10) + 15$
    $= \$69,335,000$

6-31
Green County is planning to construct a bridge across the south branch of Carey Creek to facilitate traffic flow though Clouser Canyon. The first cost for the bridge will be $9,500,000. Annual maintenance and repairs the first year of operation are estimated to be $10,000 and are expected to increase by $1,000 each year thereafter. In addition to regular maintenance, every five years the road way will be re-surfaced at a cost of $750,000 and the structure must be painted every three

years at a cost of $100,000. If Green County uses 5% as its cost of money and the bridge is expected to last 20 years, what is the EUAC?

Solution

EUAC = 9,500,000(A/P, 5%, 20) + [10,000 + 1,000(A/G, 5%, 20)]
                 + 750,000(A/F, 5%, 5)(P/A, 5%, 15)(A/P, 5%, 20)
                 + 100,000(A/F, 5%, 3)(P/A, 5%, 18)(A/P, 5%, 20)
           = $922,551