

Chapter 5

Present Worth

5-1

Emma and her husband decide they will buy \$1,000 worth of utility stocks beginning one year from now. Since they expect their salaries to increase, they will increase their purchases by \$200 per year for the next nine years. What would the present worth of all the stocks be if they yield a uniform dividend rate of 10% throughout the investment period and the price/share remains constant?

Solution

PW of the base amount (\$1,000) is: $1,000(P/A, 10\%, 10) = \$6,144.57$

PW of the gradient is: $200(P/G, 10\%, 10) = \$4,578.27$

Total PW = $6,144.57 + 4,578.27 = \$10,722.84$

5-2

It takes \$10,000 to put on the local art festival each year. Immediately *before* this year's festival, the sponsoring committee determined that it had \$60,000 in an account paying 8% interest. *After* this year, how many more festivals can be sponsored without raising more money? Think carefully!

Solution

$$\begin{aligned} 60,000 - 10,000 &= 10,000(P/A, 8\%, n) \\ (P/A, 8\%, n) &= 50,000/10,000 \\ &= 5 \end{aligned}$$

From the $i = 8\%$ table $n = 6$

This is the number of festivals after this year's. There will be some money left over but not enough to pay for a 7th year.

5-3

A scholarship is to be established that will pay \$200 per quarter at the beginning of Fall, Winter, and Spring quarters. It is estimated that a fund for this purpose will earn 10% interest, compounded quarterly. What lump sum at the beginning of Summer quarter, when deposited, will assure that the scholarship may be continued into perpetuity?

Solution

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$$i = 10/4 = 2\frac{1}{2}\%$$

$$P = 200(P/A, 2\frac{1}{2}\%, 3) = \$571.20$$

$$A' = 571.20(A/P, 2\frac{1}{2}\%, 4) = \$151.82$$

For $n = \infty$

$$P' = A'/i = 151.82/0.025 = \$6,073 \text{ deposit}$$

5-4

The winner of a sweepstakes prize is given the choice of a onetime payment of \$1,000,000 or a guaranteed \$80,000 per year for 20 years. If the value of money is 5%, which option should the winner choose?

Solution

Option 1: $P = \$1,000,000$

Option 2: $P = 80,000(P/A, 5\%, 20) = \$996,960$

Choose option 1: take the \$1,000,000 now

5-5

A tunnel to transport water through the Lubbock mountain range initially cost \$1,000,000 and has expected maintenance costs that will occur in a 6-year cycle as shown below.

End of Year:	1	2	3	4	5	6
Maintenance:	\$35,000	\$35,000	\$35,000	\$45,000	\$45,000	\$60,000

The capitalized cost at 8% interest is

- a. \$1,003,300
- b. \$1,518,400
- c. \$1,191,700
- d. \$13,018,350

Solution

Capitalized Cost = PW of Cost for an infinite time period.

First compute the Equivalent Annual Cost of the maintenance.

$$EAC = 35,000 + [10,000(F/A, 8\%, 3) + 15,000](A/F, 8\%, 6) = \$41,468.80$$

For $n = \infty$, $P = A/I$

$$\text{Capitalized Cost} = 1,000,000 + (41,468.80/0.08) = \$1,518,360.$$

The answer is b.

5-6

An engineer is considering buying a life insurance policy for his family. He currently owes \$77,500, and would like his family to have an annual available income of \$35,000 indefinitely (that is, the annual interest should amount to \$35,000 so that the original capital does not decrease).

- (a) If he assumes that any money from the insurance policy can be invested in an account paying a guaranteed 4% annual interest, how much life insurance should he buy?
- (b) If he now assumes the money can be invested at 7% annual interest, how much life insurance should he buy?

Solution

- (a) 4% interest $n = \infty$

$$A = Pi \text{ or } P = A/i = 35,000/0.04 = 875,000$$

$$\text{Total life insurance} = 77,500 + 875,000 = \$952,500$$

- (b) 7% interest $n = \infty$

$$P = A/i = 35,000/0.07 = 500,000$$

$$\text{Total life insurance} = 77,500 + 500,000 = \$577,500$$

5-7

Investment in a crane is expected to produce profit from its rental of \$15,000 the first year it is in service. The profit is expected to decrease by \$2,500 each year thereafter. At the end of six years assume the salvage value is zero. At 12% interest the present worth of the profits is nearest to

- a. \$39,350
- b. \$45,675
- c. \$51,400
- d. \$61,675

Solution

$$P = 15,000(P/A, 12\%, 6) - 2,500(P/G, 12\%, 6) = \$39,340$$

The answer is a.

5-8

The annual income from an apartment house is \$20,000. The annual expense is estimated to be \$2,000. If the apartment house can be sold for \$100,000 at the end of 10 years, how much should you be willing to pay for it now, with required return of 10%?

Solution

$$\begin{aligned} P &= (A_{\text{INCOME}} - A_{\text{EXPENSE}})(P/A, 10\%, 10) + F_{\text{SALE}}(P/F, 10\%, 10) \\ &= (20,000 - 2,000)(P/A, 10\%, 10) + 100,000(P/F, 10\%, 10) \\ &= \$149,160 \end{aligned}$$

5-9

A tax refund expected one year from now has a present worth of \$3,000 if $i = 6\%$. What is its

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present worth if $i = 10\%$?

Solution

Let x = refund value when received at the end of year 1 = $3,000(F/P, 6\%, 1)$;

$$PW = x(P/F, 10\%, 1)$$

Therefore the PW if $i = 10\% = 3,000(F/P, 6\%, 1)(P/F, 10\%, 1) = \$2,890.94$

5-10

Your company has been presented with an opportunity to invest in a project. The facts on the project are presented below:

Investment Required	\$60,000,000
Salvage Value after 10 Years	0
Gross Income	20,000,000
Annual Operating Costs:	
Labor	2,500,000
Materials, Licenses, Insurance, etc*	1,000,000
Fuel and Other Costs	1,500,000
Maintenance Costs	500,000
*Beginning of period cash flow	

The project is expected to operate as shown for ten years. If management expects to make 15% on its investments before taxes, would you recommend this project?

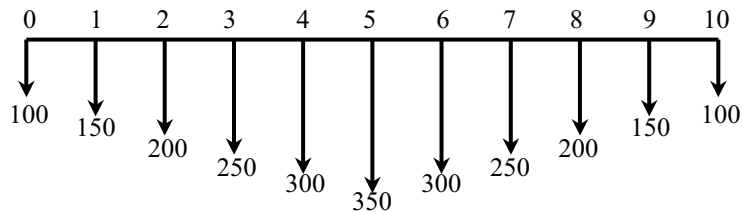
Solution

$$PW = -60,000,000 + [20,000,000 - 4,500,000 - 1,000,000(F/P, 15\%, 1)](P/A, 15\%, 10) \\ = \$12,022,650$$

Accept the project due to the positive NPW

5-11

Find the present worth of the following cash flow diagram if $i = 8\%$.



Solution

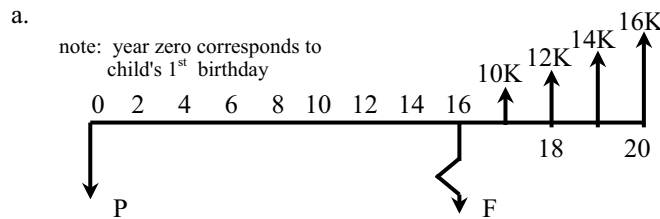
$$\begin{aligned}
 P &= 100 + 150(P/A, 8\%, 5) + 50(P/G, 8\%, 5) \\
 &\quad + [300(P/A, 8\%, 5) - 50(P/G, 8\%, 5)](P/F, 8\%, 5) \\
 &= \$1,631.97
 \end{aligned}$$

5-12

A couple wants to begin saving money for their child's education. They estimate that \$10,000 will be needed on the child's 18th birthday, \$12,000 on the 19th birthday, \$14,000 on the 20th birthday, and \$16,000 on the 21st birthday. Assume an 8% interest rate with only annual compounding. The couple is considering two methods of setting aside the needed money.

- a. How much money would have to be deposited into the account on the child's first birthday (note: a child's "first birthday" is celebrated one year after the child is born) to accumulate enough money to cover the estimated college expenses?
- b. What uniform annual amount would the couple have to deposit each year on the child's first through seventeenth birthdays to accumulate enough money to cover the estimated college expenses?

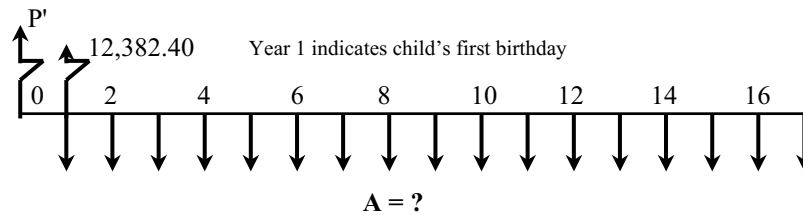
Solution



Let F = the \$'s needed at the beginning of year 16
 $= 10,000(P/A, 8\%, 4) + 2,000(P/G, 8\%, 4)$
 $= 42,420$

The amount needed today $P = 42,420(P/F, 8\%, 16) = \$12,382.40$

- b.



$P' = 12,382.40(P/F, 8\%, 1) = 11,464.86$

$A = 11,464.86(A/P, 8\%, 17) = \$1,256.55$

5-13

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Assume you borrowed \$50,000 at an interest rate of 1 percent per month, to be repaid in uniform monthly payments for 30 years. In the 163rd payment, how much of it would be interest, and how much of it would be principal?

Solution

In general, the interest paid on a loan at time t is determined by multiplying the effective interest rate times the outstanding principal just after the preceding payment at time $t - 1$.

To find the interest paid at time $t = 163$, (call it I_{163}) first find the outstanding principal at time $t = 162$ (call it P_{162}).

This can be done by computing the future worth at time $t = 162$ of the amount borrowed, minus the future worth of 162 payments. Alternately, compute the present worth, at time 162, of the 198 payments remaining.

The uniform payments are $50,000(A/P, 1\%, 360) = \514.31 , therefore

$$P_{162} = 50,000(F/P, 1\%, 162) - 514.31(F/A, 1\%, 162) = 514.31(P/A, 1\%, 198) = \$44,259.78$$

The interest $I_{163} = 0.01(44,259.78) = \442.59

The principal in the payment is $514.31 - 442.59 = \$71.72$

5-14

A municipality is seeking a new tourist attraction, and the town council has voted to budget \$500,000 for the project. A survey shows that an interesting cave can be enlarged and developed for a contract price of \$400,000. It is expected to have an infinite life. The estimated annual expenses of operation total \$50,000. The price per ticket is to be based upon an average of 12,000 visitors per year. If money is worth 8%, what should be the price of each ticket?

Solution

If the \$100,000 cash, left over after developing the cave, is invested at 8%, it will yield a perpetual annual income of \$8,000. This \$8,000 can be used toward the \$50,000 a year of expenses. The balance of the expenses can be raised through ticket sales, making the price per ticket:

$$\text{Ticket price} = \$42,000/12,000 \text{ tickets} = \$3.50$$

Alternate solution:

$$\begin{aligned} PW_{\text{COST}} &= PW_{\text{BENEFIT}} \\ 400,000 + (50,000)/.08 &= 500,000 + T/.08 \\ 400,000 + 625,000 &= 500,000 + T/.08 \\ T &= 525,000(.08) \\ &= 42,000 \end{aligned}$$

$$\text{Ticket Price} = \$42,000/12,000 \text{ tickets} = \$3.50$$

5-15

5-18

Mary Ann requires approximately 30 pounds of bananas each month, January thru June, and 35 pounds of bananas each month, July through December, to make banana cream pies for her cast-away friends the Skipper, Gilligan, the Professor, Ginger, and the Millionaire and his Wife (the Howells). Bananas can be bought at a local market for 40 cents/pound. If Mary Ann's cost of money is 3%, how much should she set aside at the beginning of each year to pay for the bananas?

- a. \$149.50
- b. \$150.50
- c. \$152.50
- d. \$153.50

Solution

$$\begin{array}{ll} \text{Cost of bananas January thru June} & 30 \times .40 = \$12 \\ \text{July thru December} & 35 \times .40 = \$14 \end{array}$$

$$i = 3/12 = 1/4\%$$

$$P = 12(P/A, 1/4\%, 6) + 14(P/A, 1/4\%, 6)(P/F, 1/4\%, 6) \\ = \$153.41$$

The answer is d.

5-19

A project has a first cost of \$10,000, net annual benefits of \$2,000, and a salvage value of \$3,000 at the end of its 10-year useful life. The project will be replaced identically at the end of 10 years, and again at the end of 20 years. What is the present worth of the entire 30 years of service if the interest rate is 10%?

Solution

$$\text{PW of 10 years} = -10,000 + 2,000(P/A, 10\%, 10) + 3,000(P/F, 10\%, 10) = \$3,445.76$$

$$\text{PW of 30 years} = 3,445.76[1 + (P/F, 10\%, 10) + (P/F, 10\%, 20)] = \$5,286.45$$

Alternate Solution:

$$\begin{aligned} \text{PW of 30 years} &= -10,000[1 + (P/F, 10\%, 10) + (P/F, 10\%, 20)] + 2,000(P/A, 10\%, 30) \\ &\quad + 3000[(P/F, 10\%, 10) + (P/F, 10\%, 20) + (P/F, 10\%, 30)] \\ &= \$5,286.45 \end{aligned}$$

5-20

The present worth of costs for a \$5,000 investment with a complex cash flow diagram is \$5,265. What is the capitalized cost if the project has a useful life of 12 years, and the MARR is 18%?

Solution

$$\text{Capitalized Cost} = 5,265(A/P, 18\%, 12)(P/A, 18\%, \infty) = \$6,102$$

5-21

A used car dealer tells you that if you put \$1,500 down on a particular car your payments will be \$190.93 per month for 4 years at a nominal interest rate of 18%. Assuming monthly compounding, what is the cost of the car?

Solution

$$A = \$190.93 \text{ per period} \quad i = 18/12 = 1.5\% \quad n = (12)(4) = 48$$

$$P = 1,500 + 190.93(P/A, 1\frac{1}{2}\%, 48) \\ = \$8,000$$

5-22

If the current interest rate on bonds of a certain type is 10%, compounded semiannually, what should the market price of a \$1,000 face value, 14% bond be? The bond will mature (pays face value) 6½ years from today and the next interest payment to the bondholder will be due in six months.

Solution

$$\text{Semi-annual interest payment} = .07(1,000) = \$70$$

$$PV = \$70(P/A, 5\%, 13) + \$1,000(P/F, 5\%, 13) = \$1,187.90$$

5-23

What is the price of a 3-year Savings Certificate worth \$5,000 three years from now, at 12 % interest, compounded continuously?

Solution

$$P = Fe^{-in} = \$5,000e^{-(0.12)3} = 5,000e^{-0.36} = \$3,488.50$$

5-24

What is the present worth of a cash flow that decreases uniformly, by \$20 per year, from \$400 in Year 11 to \$220 in Year 20, if *i* equals 10%?

Solution

$$PW = [400(P/A, 10\%, 10) - 20(P/G, 10\%, 10)](P/F, 10\%, 10) \\ = \$770.91$$

5-25

A resident will give money to his town to purchase a statue honoring the town founders and to maintain it at a cost of \$500 per year forever. If an interest rate of 10% is used, and the resident gives a total of \$15,000; how much can be paid for the statue?

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Solution

$$\begin{aligned} \text{Capitalized Cost} &= 15,000 = P + 500(P/A, 10\%, \infty) \\ P &= 15,000 - 500(1/.1) = \$10,000 \end{aligned}$$

5-26

A project being considered by the XYZ Company will have \$100,000 in construction costs each of the first three years of the project. Income of \$100,000 will begin flowing in year four and continue through year 10. The net present worth at 4% of the project is

- \$184,841
- \$188,513
- \$251,089
- \$256,078

Solution

$$\begin{aligned} P &= -100,000(P/A, 4\%, 3) + 100,000(P/A, 4\%, 7)(P/F, 4\%, 3) \\ &= \$256,077.80 \end{aligned}$$

The answer is d.

5-27

Corns Squeezings Inc. is considering the purchase of new mashing equipment. The CEO of the company, Dollar Signs Dallas, has requested that you provide the net present value for the equipment being considered. The relevant data for the new equipment are presented below:

First Cost	\$125,000	
Annual Income	10,000	
Annual Operating Costs	8,000	The first year and increasing by \$750 per year
Property Taxes	5% of first cost	Payable at the end of each year
Salvage Value	8% of first cost	

CSI has a minimum attractive rate of return (MARR) of 4% and the equipment is has an expected useful life of 6 years.

Solution

<u>Yr</u>		
0	First Cost	(125,000)
1-6	Annual Income 10,000(P/A, 4%, 6)	52,420
1-6	Annual Costs 8,000(P/A, 4%, 6) + 750(P/G, 4%, 6)	(51,316)
1-6	Taxes 6,250(P/A, 4%, 6)	(32,763)
6	Salvage Value 10,000(P/F, 4%, 6)	<u>7,903</u>
	NPV	\$(148,756)

5-28

Dolphin Inc. trains mine seeking dolphins in a 5-mine tank. They are considering purchasing a new tank. The U.S. Navy will pay \$105,000 for each dolphin trained and a new tank costs \$750,000 and each realistic dummy mine costs \$250,000. The new tank will allow the company to train 3 dolphins per year and will last 10 years costing \$50,000 per year to maintain. Determine the net present value if the MARR equals 5%?

Solution

$$\begin{aligned} \text{NPV} &= -\text{Cost} - \text{Cost of Mines} - \text{Annual Maintenance}(P/A, 5\%, 10) + \text{Income}(P/A, 5\%, 10) \\ &= -750,000 - 250,000(5) - 50,000(P/A, 5\%, 10) + 105,000(3)(P/A, 5\%, 10) \\ &= \$46,330 \end{aligned}$$

5-29

Consider two investments:

- (1) Invest \$1000 and receive \$110 at the end of each month for the next 10 months.
- (2) Invest \$1200 and receive \$130 at the end of each month for the next 10 months.

If this were your money, and you want to earn at least 12% interest on your money, which investment would you make, if any? What nominal interest rate do you earn on the investment you choose? Solve by present worth analysis.

Solution

$$i = 12/12 = 1\%$$

$$\begin{aligned} \text{Alternative 1: } \text{NPW} &= 110(P/A, 1\%, 10) - 1000 = \$41.81 \\ \text{Alternative 2: } \text{NPW} &= 130(P/A, 1\%, 10) - 1200 = \$31.23 \end{aligned}$$

Choose Alternative 1 → Maximum NPW

$$\begin{aligned} \text{Nominal Interest: } \text{NPW} = 0 &= -1000 + 110(P/A, i\%, 10) \\ (P/A, i\%, 10) &= 9.1 \end{aligned}$$

From the interest tables $i \approx 1.75\%$

$$\text{Nominal interest} = 1.75\% \times 12 \text{ mo.} = 21\%$$

5-30

The lining of a chemical tank in a certain manufacturing operation is replaced every 5 years at a cost of \$5,000. A new type lining is now available that would last 10 years, but costs \$9,000. The tank needs a new lining now and you intend to use the tank for 40 years, replacing linings when necessary. Compute the present worth of costs of 40 years of service for the 5-year and 10-year linings if $i = 10\%$.

Solution

PW 5 yr Lining:

$$\text{PW} = [5,000(A/P, 10\%, 5)](P/A, 10\%, 40) = \$12,898.50$$

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PW 10 yr Lining:

$$PW = [9,000(A/P, 10\%, 10)](P/A, 10\%, 40) = \$14,319.39$$