

Chapter 18

Uncertainty and Probability

18-1

Tee-to Green Golf Inc. is considering the purchase of new automated club assembly equipment. The industrial engineer for TGG thinks that she has determined the “best” choice. However she is uncertain how to evaluate the equipment because of questions concerning the actual annual savings and salvage value at the end of the expected life. The equipment will cost \$500,000 and is expected to last eight years. Information concerning the savings and salvage value estimates and the projected probabilities is presented below:

	$p = .20$	$p = .50$	$p = .25$	$p = .05$
Savings per year	\$65,000	\$82,000	\$90,000	\$105,000
Salvage value	40,000	55,000	65,000	75,000

Determine the NPW if TGG’s MARR is 6%.

Solution

$$E(\text{Savings}) = .2(65,000) + .5(82,000) + .25(90,000) + .05(105,000) = \$81,750$$

$$E(\text{Salvage}) = .2(40,000) + .5(55,000) + .25(65,000) + .05(75,000) = \$55,500$$

$$\begin{aligned} \text{NPW} &= -500,000 + 81,750(P/A, 6\%, 8) + 55,500(P/F, 6\%, 8) \\ &= \$42,489 \end{aligned}$$

18-2

Acme Insurance offers an insurance policy that pays \$1,000 for lost luggage on a cruise. Historically the company pays this amount in 1 out of every 200 policies it sells. What is the minimum amount Acme must charge for such a policy if they desire to make at least \$10 dollars per policy?

Solution

$$\text{The probability that a loss occurs is } \frac{1}{200} = .005$$

$$\text{The expected loss to the company is therefore } .005(1,000) = \$5$$

To make a profit of \$10 from each policy sold, they must charge \$15 per policy

18-3

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Consolidated Edison Power is evaluating the construction of a new electric generation facility. The two choices are a coal burning plant (CB) or a gaseous diffusion (GD) plant. The CB plant will cost \$150 per megawatt to construct, and the GD plant will cost \$300 per megawatt. Due to uncertainties concerning fuel availability and the impact of future air- and water- quality regulations, the useful life of each plant is unknown, but the following probability estimates have been made

Useful life (years)	Probability	
	CB plant	GD Plant
10	.10	.05
20	.50	.25
30	.30	.50
40	.10	.20

- Determine the expected life of each plant.
- Based on the ratio of construction cost per megawatt to expected life, which plant would you recommend Con Ed build?

Solution

- Expected life

$$\text{Coal burning} = .10(10) + .50(20) + .30(30) + .10(40) = 24 \text{ years}$$

$$\text{Gaseous diffusion} = .05(10) + .25(20) + .50(30) + .20(40) = 28.5 \text{ years}$$

- Ratios

$$\text{Coal burning} = 150/24 = \$6.25 \text{ per megawatt per year}$$

$$\text{Gaseous diffusion} = 300/28.5 = \$10.53 \text{ per megawatt per year}$$

Recommend the coal burning plant

18-4

Crush Cola Company must purchase a bottle capping machine. Information concerning the machine and possible cash flows is presented below.

	$p = .30$	$p = .50$	$p = .20$
First Cost	\$40,000	\$40,000	\$40,000
Annual Savings	2,000	3,500	5,000
Annual Costs	7,000	5,000	4,000
Actual Salvage Value	4,000	5,000	6,500

The machine is expected to have a useful life of 10 years. Crush has a MARR of 6%. Determine the NPW of the machine.

Solution

$$\begin{aligned} E(\text{Saving/Costs}) &= (2,000 - 7,000)(.30) + (3,500 - 5,000)(.50) + (5,000 - 4,000)(.20) \\ &= -\$2,050 \end{aligned}$$

$$E(\text{Salvage Value}) = 4,000(.30) + 5,000(.50) + 6,500(.20) \\ = \$5,000$$

$$\text{NPW} = -40,000 - 2,050(P/A, 6\%, 10) + 5,000(P/F, 6\%, 10) = -\$52,296.00$$

18-5

The two finalists in a tennis tournament are playing for the championship. The winner will receive \$60,000 and the runner-up \$35,000. Determine the expected winnings for each participant if the players are considered to be evenly matched. What would the expected winnings be if one player were favored by 4 to 1 odds.

Solution

Evenly matched, both players expected winnings will be the same.

$$\text{Winnings} = .5(60,000) + .5(35,000) = \$47,500$$

Assume player A is favored by 4 to 1 odds. The probability that A wins is then 4/5.

$$\text{Player A's expected winnings} = .8(60,000) + .2(35,000) = \$55,000$$

$$\text{Player B's expected winnings} = .2(60,000) + .8(35,000) = \$40,000$$

18-6

A roulette wheel consists of 18 black slots, 18 red slots, and 2 green slots. If a \$100 bet is placed on black, what is the expected gain or loss? (A bet on black or red pays even money.)

Solution

$$\text{The probability of black occurring} = \frac{18}{38}$$

$$\text{Expected value of the bet} = 100 \frac{18}{38} - 100 \frac{20}{38} = \frac{100}{19} \approx \$5.26 \text{ loss}$$

18-7

Palmer Potatoes Chips Inc. must purchase new potato peeling equipment for its Martin, Tennessee plant. The plant engineer has determined there are three possible set-ups that can be purchased. Relevant data are presented below. All machines are expected to be used six years and PPC Inc.'s MARR is 10%. Which machine should be chosen?

<u>Naked Peel</u>			
First Cost	\$45,000		
Annual Costs		p = .2	\$3,000

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		p = .7	4,500
		p = .1	5,500
Salvage Value		p = .7	\$7,500
		p = .3	9,500
<u>Skinner</u>			
First Cost	\$52,000		
Annual Costs		p = .4	\$5,000
		p = .4	6,500
		p = .2	8,500
Salvage Value		p = .4	\$5,500
		p = .3	7,500
		p = .3	8,500
<u>Peel-O-Matic</u>			
First Cost	\$76,000		
Annual Costs		p = .3	\$5,000
		p = .5	7,500
		p = .2	9,500
Salvage Value		p = .6	\$8,500
		p = .4	9,000

Solution

$$NPV = -\text{First Cost} + \text{Annual Savings}(P/A, 10\%, 6) + \text{Salvage Value}(P/F, 10\%, 6)$$

Naked Peel

$$E(\text{Annual costs}) = .2(3,000) + .7(4,500) + .1(5,500) = \$4,300$$

$$E(\text{Salvage value}) = .7(7,500) + .3(9,500) = \$8,100$$

$$NPW = -\$59,154$$

Skinner

$$E(\text{Annual costs}) = \$6,300$$

$$E(\text{Salvage value}) = \$7,000$$

$$NPW = -\$75,485$$

Peel-O-Matic

$$E(\text{Annual costs}) = \$7,150$$

$$E(\text{Salvage value}) = \$8,700$$

$$NPW = -\$102,227$$

Fixed output, therefore choose lowest NPW, Naked Peel.

18-8

Krispy Kookies is considering the purchase of new dough mixing equipment. The estimated NPW and probabilities of the four possible outcomes are presented in the table below. Calculate the expected annual worth of the equipment if the life of the equipment is six years and $i = 8\%$.

<u>Outcome</u>	<u>NPW</u>	<u>Probability</u>
1	\$34,560	.15
2	38,760	.25
3	42,790	.40
4	52,330	.20

Solution

$$E(\text{NPW}) = .15(34,560) + .25(38,760) + .40(42,790) + .20(52,330) = \$42,456$$

$$AW = 42,456(A/P, 8\%, 6) = \$9,183.23$$

18-9

A new heat exchanger must be installed by CSI Inc. Alternative A has an initial cost of \$33,400 and alternative B has an initial cost of \$47,500. Both alternatives are expected to last ten years. The annual cost of operating the heat exchanger depends on ambient temperature in the plant and energy costs. The estimate of the cost and probabilities for each alternative is presented below. If CSI has a MARR of 8% and uses rate of return analysis for all capital decisions, which exchanger should be purchased?

	<u>Annual Cost</u>	<u>Probability</u>
Alternative A	\$4,500	.10
	7,000	.60
	8,000	.25
	9,250	.05
Alternative B	\$4,000	.20
	5,275	.60
	6,450	.15
	8,500	.05

SolutionAlternative A

$$E(\text{Annual Cost}) = .10(4,500) + .60(7,000) + .25(8,000) + .05(9,250) = \$7,112.50$$

Alternative B

$$E(\text{Annual Cost}) = .20(4,000) + .60(5,275) + .15(6,450) + .05(8,500) = \$5,357.50$$

Incremental Analysis is required

B-A

$$\text{NPW} = 0 \text{ at IRR}$$

$$0 = (-47,500 - (-33,400)) + (-5,375.50 - (-7,112.50))(P/A, i\%, 10)$$

$$0 = -14,100 + 1,737(P/A, i\%, 10)$$

$$(P/A, i\%, 10) = 8.12$$

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$$i = 4\% \quad P/A = 8.111$$

$$IRR \approx 4\%$$

CSI should purchase the least expensive alternative, A

18-10

Northeast Airlines is considering entering a bid for a new Asian route. The route is expected to command a rather large price. There is uncertainty associated with all elements of the investment. The winner of the route will have use of it for five years before the bidding process is repeated. Information developed by Northeast concerning estimated cash flows and probabilities is summarized below. Based on a MARR of 8% determine the expected net present worth of the route.

Element	p = .15	p = .45	p = .30	p = .10
Bid Amount	11,500,000	22,250,000	27,500,000	38,250,000
Net Annual Income	3,000,000	5,700,000	7,000,000	9,600,000

Solution

$$NPW 1 = -11,500,000 + 3,000,000(P/A, 8\%, 5) = \$479,000$$

$$NPW 2 = -22,250,000 + 5,700,000(P/A, 8\%, 5) = \$510,100$$

$$NPW 3 = -27,500,000 + 7,000,000(P/A, 8\%, 5) = \$451,000$$

$$NPW 4 = -38,250,000 + 9,600,000(P/A, 8\%, 5) = \$ 82,800$$

$$E(NPW) = .15(479,000) + .45(510,100) + .30(451,000) + .10(82,800) = \$444,975$$

18-11

The probability that a machine will last a certain number of years is given in the following table.

<u>Years of Life</u>	<u>Probability of Obtaining Life</u>
10	0.15
11	0.20
12	0.25
13	0.20
14	0.15
15	0.05

What is the expected life of the machine?

Solution

$$\begin{aligned} \text{Expected value} &= 10(0.15) + 11(0.20) + 12(0.25) + 13(0.20) + 14(0.15) + 15(0.05) \\ &= 12.15 \text{ years} \end{aligned}$$

18-12

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In the game of craps two die are tossed. One of the many bets available is the “Hard-way 8”. A \$1 bet will return to the bettor \$4 if in the game the two die come up 4 and 4 prior to one of the other ways of totaling eight. For the \$1 bet, what is the expected result?



- a. \$.11
- b. \$.33
- c. \$.66
- d. \$.80

Solution

There are five ‘ways’ of rolling an eight.→

<u>Die 1</u>	<u>Die 2</u>
2	6
3	5
4	4
5	3
6	2

Hard-way 8 probability = 1/5

$$E(\$) = 1/5(\$4) + 4/5(\$0) = \$.80$$

The answer is d.

18-13

A dam is being considered to reduce river flooding in the Forked River Basin. Information concerning the possible alternatives is presented below.

<u>Dam Height (H)</u>	<u>First Cost</u>	<u>Annual Probability of flood if Height =</u>	<u>\$ Damages If Flooding Occurs</u>
0	\$ 0	0.25	\$800,000
20'	700,000	0.05	500,000
30'	800,000	0.01	300,000
40'	900,000	0.002	200,000

Which dam height minimizes the expected total annual cost? The state uses an interest rate of 5% for flood protection projects, and all dams must last 50 years.

Solution

H = 0 (No dam is built)

$$EUAC = 800,000(.25) = \$200,000$$

H = 20'

$$EUAC = 700,000(A/P, 5\%, 50) + 500,000(.05) = \$63,360$$

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$$H = 30'$$

$$\begin{aligned} \text{EUAC} &= 800,000(\text{A/P}, 5\%, 50) + 300,000(.01) \\ &= \$46,840 \end{aligned}$$

$$H = 40'$$

$$\begin{aligned} \text{EUAC} &= 900,000(\text{A/P}, 5\%, 50) + 200,000(.002) \\ &= \$49,720 \end{aligned}$$

In order to minimize annual cost, build 30-foot dam.