Chapter 17

Sensitivity Analysis

17-1

A coal burning power plant has been ordered by the government to install a $5 million pollution abatement device to remove sulphur that is currently being emitted into the air. The sulphur is removed by allowing the plant's exhaust to pass through a filter. The filtration system requires the presence of a certain chemical. The purchase price of the chemical is $1,000 per kilogram. Studies have been conducted that show that the number of units of sulphur that may be recovered annually from the exhaust is equal to 100 times the square root of the number of kilograms of the chemical used in the filtration system.

Therefore:

$$(\text{units of sulphur}) = 100 \times (\text{kg of chemical})^{1/2}$$

Each unit of sulphur that is removed may then be sold by the power plant to chemical supply companies for $300. The filtration system and chemical have an expected life of 20 years at which time the chemical will have a resale value of $500 per kilogram, while the filtration system itself has no resale value.

Using a before-tax minimum attractive rate of return (MARR) of 10%, find the optimal amount of the chemical that should be purchased by the power plant.

Solution

Let X = number of kg of chemical purchased

Net Annual Cost (X) = (purchase cost of pollution abatement device)(A/P, 10%, 20)
+ (chemical purchase cost)(A/P, 10%, 20)
- (salvage value of chemical)(A/P, 10%, 20)
- (annual sale value of sulphur)

Net Annual Cost (X) = (5,000,000)(A/P, 10%, 20) + (1,000X)(A/P, 10%, 20)
- (500X)(A/F, 10%, 20) - (300) $(100\sqrt{X})$
$= 587.5 + 108.75X - 30{,}000\sqrt{X}$

To minimize cost differentiate Net Annual Cost (X) and set = 0

$$\frac{d\text{NAC(X)}}{d\text{X}} = 108.75 - \frac{30{,}000}{2\sqrt{\text{X}}} = 0$$

$$\frac{1}{\sqrt[2]{X}} = \frac{108.75}{30,000} \Rightarrow \sqrt{X} = \frac{30,000}{2(108.75)} = 137.931$$

$X = 137.931^2 = 19,024.97$

Check for max. or min. $\frac{d^2 NAC(X)}{dX^2} = \frac{7,500}{\sqrt{X^3}} > 0$ For $X > 0$

17-2

The annual income from an apartment house is $20,000. The annual expense is estimated to be $2000. If the apartment house can be bought today for $149,000, what is the breakeven resale price in ten years with 10% considered a suitable interest rate?

Solution

$$P = (A_{INCOME} - A_{EXPENSES})(P/A, i\,\%, n) + F_{RE\text{-}SALE}(P/F, i\,\%, n)$$
$$149,000 = (20,000 - 2,000)(P/A, 10\%, 10) + F_{RE\text{-}SALE}\,(P/F, 10\%, 10)$$
$$149,000 = 18,000(6.145) + F_{RE\text{-}SALE}\,(.3855)$$
$$F_{RE\text{-}SALE} = \$99,584.95$$

17-3

Oliver Douglas decides to install a fuel storage system for his farm that will save him an estimated 6.5 cents/gallon on his fuel cost. Initial cost of the system is $10,000 and the annual maintenance is a uniform gradient amount of $25. After a period of 10 years the estimated salvage is $3,000. If money is worth 12%, what is the breakeven quantity of fuel?

Solution

EAC =(10,000 - 3,000)(A/P, 12%, 10) + 3,000(.12) + 25(A/G, 12%, 10)
= $1,688.63

EAB = Gallons(.065) = $G(.065)

0 = -1,688.63 + G(.065)
G = 25,979 gallons

17-4

A land surveyor just starting in private practice needs a van to carry crew and equipment. He can lease a used van for $8,000 per year, paid at the beginning of each year, in which case maintenance is provided. Alternatively, he can buy a used van for $12,000 and pay for maintenance himself. He expects to keep the van three years at which time he expects to sell it for $3,500. What is the most he should pay for uniform annual maintenance to make it worthwhile buying the van instead of leasing it, if his MARR is 10%?

Solution

Lease:

EUAC = 6,500(F/P, 10%, 1) = 6,500(1.10) = 7,150

Buy:

EUAC = 16,000(A/P, 10%, 3) + M - 3,000(A/F, 10%, 3)

Setting equal and solving for M

7,150 = 6,433.60 + M - 906.30
M = $1,622.70

17-5

The investment in a crane is expected to produce profit from its rental as shown below, over the next six years. Assume the salvage value is zero. Assuming 12% interest, what is the breakeven cost of the crane?

Year	Profit
1	$15,000
2	12,500
3	10,000
4	7,500
5	5,000
6	2,500

Solution

PW_{PROFIT} = 15,000(P/A, 12%, 6) - 2,500(P/G, 12%, 6) = $39,340
$Cost_{BE}$ = $39,340

17-6

ABC Manufacturing has a before-tax minimum attractive rate of return (MARR) of 12% on new investments. What uniform annual benefit would Investment B have to generate to make it preferable to Investment A?

Year	Investment A	Investment B
0	- $60,000	- $45,000
1 - 6	+15,000	?

Solution

NPW of A = - 60 + 15(P/A, 12%, 6) = 1.665

NPW of B ≥ 1.665 = - 45 + B(P/A, 12%, 6)
∴ B = 11,351

B > $11,351 per year

17-8

Data for two drill presses under consideration by B&R Gears are listed below. Assuming an interest rate of 12%, what salvage value of press B will make the two alternatives equal?

	A	B

First cost	$30,000	$36,000
Annual maintenance	1,500	2,000
Salvage value	5,000	?
Useful life	6 years	6 years

Solution

EAC = P(A/P, i %, n) - S(A/F, i %, n) + Other Costs

DRILL PRESS A:

EAC = 30,000(A/P, 12%, 6) - 5,000(A/F, 12%, 6) + 1,500
= $8,180

DRILL PRESS B:

EAC = 36,000(A/P, 12%, 6) - SV(A/F, 12%, 6) + 2,000
= $10,755.20 - SV(.1232)

Setting the two EAC equal

8,180 = 10,755.20 - SV(.1232)
SV = $20,903

17-9

Dolphin Inc. trains mine seeking dolphins in a 5-mine tank. They are considering purchasing a new tank. A new tank costs $750,000 and realistic dummy mines cost $250,000. The new tank will allow the company to train 3 dolphins per year and will last 10 years costing $50,000 per year to maintain. How much must Dolphin Inc. receive (per dolphin) from the Navy in order to breakeven if the MARR equals 5%?

Solution

NPV = -Cost - Cost of Mines - Annual Maintenance(P/A, 5$, 10) + Income(P/A, 5%, 10)
= -750,000 - 250,000(5) - 50,000(P/A, 5%, 10) + 3(x)(P/A, 5%, 10)
x = $103,000