Chapter 15

Inflation

15-1
The first sewage treatment plant for Athens, Georgia cost about $2 million in 1964. The utilized capacity of the plant was 5 million gallons/day (mgd). Using the commonly accepted value of 135 gallons/person/day of sewage flow, find the cost per person for the plant. Adjust the cost to 1984 dollars with inflation at 6%. What is the annual capital expense per person if the useful life is 30 years and the value of money is 10%?

Solution

Population equivalents = 5 mgd/135 = 37,037

$$\text{Cost per capita} = \frac{\$2{,}000{,}000}{37{,}037} = \$54$$

$1984_{\$}$, F = 54(F/P, 6%, 20) = $173.18

Annual Cost, A = 173.18(A/P, 10%, 30) = $18.37

15-2
How much life insurance should a person buy if he wants to leave enough money to his family, so they receive $25,000 per year in interest, of consent Year 0 value dollars? The interest rate expected from banks is 11%, while the inflation rate is expected to be 4% per year.

Solution

The actual (effective) rate that the family will be getting is

$$i' = \frac{i-f}{1+f} = \frac{0.11-0.04}{1.04} = 0.0673 = 6.73\%$$

To calculate P, $n = \infty$ (capitalized cost)

$$P = \frac{A}{i'} = \frac{25{,}000}{0.0673} = \$371{,}471$$

Therefore, he needs to buy about $371,500 of life insurance.

15-3

A European investor lives near to one of his country's borders. In Country A (where he lives), an 8% interest rate is offered in banks, and the inflation rate is 3%. Country B, on the other hand, has an inflation rate of 23%, and banks are offering 26% interest on deposits.

a) What is the real or effective interest rate that this person gets when investing in his Country?
b) This investor believes that the currency of Country B will not change in its value relative to the value of the currency of Country A during this year. In which country would he get a larger effective interest rate?
c) Suppose that he invests in a bank in Country B, and that his prediction was wrong. The currency of Country B was devaluated 20% with respect to the exchange value of Country A's currency. What is the effective interest rate that he obtained?

Solution

a) i' = ? if i = 80%, f = 3%

$$i = i' + f + i'F$$
$$.08 = i' + .03 + i'(.03)$$
$$i' = 0.0485$$
$$= 4.85\%$$

b) if investment in Country A: $i'_A = 0.0485$
if investment in Country B: $i_B = 26\%$, $f_A = 3\%$
(note that he lives in Country A. Inflation of Country B does not affect him directly)

$$i'_B = \frac{i_B - f_A}{1+f_A} = \frac{0.26-0.03}{1+0.03} = 0.2233 = 22.33\%$$

He can get a larger effective interest rate in Country B.

c) Let X = amount originally invested in B (measured in currency A).

The amount collected at end of 1 year (measured in currency A) =

$$\underbrace{(1.0 - 0.2)}_{\text{Due to the devaluation}} \quad \underbrace{(1.26)}_{\text{Due to initial deposit (+) interest}} = 1.008X$$

the interest is then $i = \frac{1.008X - X}{X} = 0.008$

but during the year the inflation in Country A (where he lives) was 3%, therefore

i = 0.008
f = 0.03
i' = ?

$$i' = \frac{0.008-0.03}{1+0.03} = -0.02136$$

He actually lost money (negative effective interest rate of - 2.136%).

15-4

Property, in the form of unimproved land, is purchased at a cost of $8,000 and is held for six years when it is sold for $32,600. An average of $220 each year is paid in property tax and may be accounted for at an interest of 12%. The income tax rate on the long term capital gain is 15% of

the gain. Inflation during the period is 7% per year. What is the annual rate of return for this investment?

Solution

Long term gains = 32,600 - 8,000 = 24,600
Tax on long-term gain = .15 x 24,600 = 3,690
Property tax = 220(F/A, 12%, 6) = 1,785.30

Adjusted FW = 32,600 - 3,690 - 1785.30 = 27,624.70
also FW = $8{,}000(1 + i_{eq})^6$

$$\therefore \left(1 + i_{eq}\right) = \left(\frac{27{,}124.70}{8{,}000}\right)^{\frac{1}{6}} = 1.2257$$

$$\left(1 + i_{eq}\right) = \left(1 + i\right)\left(1 + i_f\right)$$

$$1 + i = \frac{1.2257}{1.07} = 1.1455 \text{ or } 14.6\% \text{ rate of retrun}$$

15-5

The auto of your dreams costs $20,000 today. You have found a way to earn 15% tax free on an "auto purchase account". If you expect the cost of your dream auto to increase by 10% per year, how much would you need to deposit in the "auto purchase account" to provide for the purchase of the auto 5 years from now?

Solution

Cost of auto 5 years hence: $F = P(1 + \text{inflation rate})^n = 20{,}000\ (1 + 0.10)^5 = \$32{,}210$

Amount to deposit now to have $32,210 five years hence

P = F(P/F, i%, n) = 32,210(P/F, 15%, 5) = $16,014.81

15-6

On January 1, 1975 the National Price Index was 208.5, and on January 1, 1985 it was 516.71. What was the inflation rate, compounded annually, over that 10-year period? If that rate continues to hold for the next 10 years, what National Price Index can be expected on January 1, 1995?

Solution

Set NPW = 0

$0 = -208.5 + 516.71(P/F, i_f, 10)$

$(P/F, i_f, 10) = \frac{208.5}{516.71}$

$= 0.4035$

From interest tables the P/F factor at 9% = 0.4224
10% = 0.3855 $\therefore$ $9\% < i_f < 10\%$

By interpolation $i_f = 9.51\%$

National Price Index$_{1995}$ = $516.71(1 + 0.0951)^{10} = 1{,}281.69$

15-7
An electronics store offers two options to buy a new laptop computer that has a price of $440.00. A customer can either pay cash and immediately receive a discount of $49.00 or she can pay for the computer on an installment plan. The installment plan has a nominal rate of 12% compounded bi-yearly and requires an initial down payment of $44.00 followed by four equal payments (principle and interest) every six months for two years.

If for the typical customer the cost of money is 5%, what is the maximum effective annual inflation rate for the next two years that would make paying cash preferred to paying installments? All figures above are quoted in time zero dollars.

Solution

The monthly payments in nominal dollars if the installment plan was selected would be

$(-440 + 44)(A/P, 6\%, 4) = -\$114.28$

The breakeven inflation rate is that such that

$NPV_{BUY} = NPV_{INSTALL}$ or $NPV_{BUY - INSTALL} = 0$

$NPV_{B-I} = ((-440 + 49) + 44) + 114.28(P/A, i_{1/2}, 4) = 0$

$(P/A, i_{1/2}, 4) = 3.0364$ therefore the nominal effective semi-annual cost of money would have to be $i_{1/2} = .115$. The nominal effective annual rate would be $i = (1.115)^2 - 1 = 0.2432$

The effective annual inflation rate can now be computed from the formula
$(1.2432) = (1.05)(1 + f)$
$f = .1840$

15-8
An automobile that cost $19,500 in 2004 has an equivalent model four years later in 2008 that cost $22,250. If inflation is considered the cause of the increase, what was the average annual rate of inflation?

Solution

$F = P(1 + i_f)^n$
$22{,}250 = 19{,}500(1 + i_f)^4$

$$\frac{22,250}{19,500} = (1 + i_f)^4$$
$$1 + i_f = (1.141)^{1/4}$$
$$1 + i_f = 1.0335$$
$$i_f = 3.35\%$$

15-9
A machine has a first cost of $100,000 (in today's dollar) and a salvage value of $20,000 (in then current dollars) at the end of its ten year life. Each year it will eliminate one full-time worker. A worker costs $30,000 (today's dollars) in salary and benefits. Labor costs are expected to escalate at 10% per year. Operating and maintenance costs will be $10,000 per year (today's dollars) and will escalate at 7% per year.

Construct a table showing before-tax cash flows in current dollars, and in today's dollars. The inflation rate is 7%.

Solution

End of	Current Dollars				Today's
Year	Savings	O & M	Capital	Total	Dollar's
0			-100,000	-100,000	-100,000
1	33,000	-10,700		22,300	20,841
2	36,300	-11,449		24,851	21,706
3	39,930	-12,250		27,680	22,595
4	43,923	-13,108		30,815	23,509
5	48,315	-14,026		34,290	24,448
6	53,147	-15,007		38,140	25,414
7	58,462	-16,058		42,404	26,407
8	64,308	-17,182		47,126	27,428
9	70,738	-18,385		52,354	28,477
10	77,812	-19,672	20,000	78,141	39,723

15-10
A project has been analyzed assuming 6% inflation and is found to have a monetary internal rate of return (IRR) of 22%. What is the real IRR for the project?

Solution

Real IRR = (1.22)/(1.06) - 1 = 0.1509 or 15.09%

15-11
A company requires a real MARR of 12%. What monetary MARR should they use if inflation is expected to be 7%?

Solution

Monetary MARR = (1.12)(1.07) - 1 = 0.1984 or 19.84%

15-12
The real interest rate is 4%. The inflation rate is 8%. What is the apparent interest rate?

Solution

$i = i' + f + i'f$
$= 0.04 + 0.08 + 0.04(0.08) = 12.32\%$

15-13
A lot purchased for $4,500 is held for five years and sold for $13,500. The average annual property tax is $45 and may be accounted for at an interest rate of 12%. The income tax rate on the long term capital gain is 15% of the gain. What is the rate of return on the investment if the allowance for inflation is treated at an average annual rate of 7%?

Solution

Long term gain = 13,500 - 4,500 = 9,000
Tax on long term gain = (.15)(9,000) = 1,350
Property tax = 45(F/A, 12%, 5) = 285.89

Adjusted FW = 13,500 - 1,350 - 285.89 = 11,864.12
also FW = $4,500(1 + i_{eq})^5$

$$\therefore \left(1 + i_{eq}\right) = \left(\frac{11,864.12}{4,500}\right)^{\frac{1}{5}} = 1.214$$

$$\left(1 + i_{eq}\right) = (1 + i)\left(1 + i_f\right)$$

$$1 + i = \frac{1.214}{1.075} = 1.129 \text{ or } 12.9\% \text{ rate of return}$$

15-14
A solar energy book gives values for a solar system as follows: initial cost, $6,500; initial fuel savings, $500/year; expected life, 15 years; value of money, 10%; inflation, 12%; and incremental income tax rate, 25%. If we define the payback condition as the time required for the present worth of the accumulated benefit to equal the accumulated present worth of the system cost, what is the time required to reach the payback condition? Since the income tax benefit is related to the annual interest expense, treat it as a reduction of the annual cost.

Solution

Annualizing P: A = 6,500(A/P, 10%, 15)
= $854.75

$1 + i_C = (1.10)(1 + 0.25 \times 0.10) = 1.1275$

PW of costs = 854.75(P/A, 12.75%, 15) = 5,595.82

$$1 + i_{eq} = \frac{1 + i_f}{1 + i} = \frac{1.12}{1.10} = 1.018$$

The solution strategy is to find the time for the PW of benefits to equal PW of cost. When the combined effect of the two rates on a distributed A amount are opposed then the net effect retains the direction of the longer rate. The inflation rate is greater than the time value of money, which is abnormal. To solve this problem, find the PW of benefit, and to do that we must get FW of the equivalent rate, i_{eq}.

Try 10 years: FW = 500(F/A, 1.8%, 10) = 500 (10.850) = $5,425.06
Try 11 years: FW = 500(F/A, 1.8%, 11) = 500 (12.045) = $6,022.72

10 years < Payback < 11 years By interpolation payback = 10.3 years

15-15

Compute the internal rate of return based on constant (Year 0) dollars for the following after-tax cash flow given in current or actual dollars. Inflation is assumed to be 7% per year. (Round to the nearest dollar.)

Year	After Tax Cash Flow In Actual Dollars
1998 (Yr 0)	-$10,000
1999	3,745
2000	4,007
2001	4,288
2002	4,588

Solution

Year	After Tax Cash Flow In Constant Dollars
1998 (0)	-$10,000
1999 (1)	$3,745(1.07)^{-1} = 3,500$
2000 (2)	$4,007(1.02)^{-2} = 3,500$
2001 (3)	$4,288(1.07)^{-3} = 3,500$
2002 (4)	$4,588(1.07)^{-4} = 3,500$

NPW = 0 at IRR

$$0 = -10,000 + 3,500(P/A, i\%, 4)$$
$$(P/A, i\%, 4) = 10,000/3,500$$
$$= 2.857$$

Searching the interest tables where n = 4, i = IRR = 15%

15-16
The capital cost of a wastewater treatment plant for a small town of 6,000 people was estimated to be about $85/person in 1969. If a modest estimate of the rate of inflation is 5.5% for the period to 1984, what is the per capita capital cost of the treatment plant in 1984?

Solution

$F = P(1+ i_f)^n$
$= 85(1+.055)^{15}$
$= 85(2.232)$
$= \$189.76$

15-17
Minor Oil Co. owns several gas wells and is negotiating a 10-year contract to sell the gas from these wells to Major Oil Co. They are negotiating on the price of the gas the first year, per thousand cubic feet (KCF), and on the escalation clause, the percentage rate of increase in the price every year thereafter. Minor expects the wells to produce 33,000 KCF the first year and to decline at the rate of 18% every year thereafter. Minor has agreed to spend $500,000 now to lay pipelines from each well to Major's nearby refinery. What should the minimum price be the first year and what should the escalation rate be if Minor wants their revenue each year to remain constant (uniform) over the life of the contract. Assume an end-of-year convention and a minimum attractive rate of return (MARR) of 15%.

Solution

Required annual income to earn the 15% MARR on $500,000:

EAB = 500,000(A/P, 15%, 10) = $99,650.

First year price = $99,650/33,000 = $3.02/KCF

Annual production declines (1 - 0.18) of initial rate each year.

Let f = required annual escalation rate

Then (1 - 0.18)(1+f) = 1 to keep the revenue constant

$$f = \frac{1}{(1-0.18)} - 1$$
$$= 0.2195/\text{year}$$

15-18
Jack purchases a lot for $40,000 cash and plans to sell it after 5 years. What should he sell it for if he wants a 20% before-tax rate of return, after taking the 5% annual inflation rate into account?

Solution

F = 40,000(F/P, 20%, 5)(F/P, 5%, 5)

= $126,988

15-19

Undeveloped property near the planned site of an interstate highway is estimated to be worth $48,000 in six years when the construction of the highway is completed. Consider a 15% capital gains tax on the gain, an annual property tax of 0.85% of the purchase price, an annual inflation rate of 7%, and an expected return of 15% on the investment. What is the indicated maximum purchase price now?

Solution

Let X = purchase cost

$1 + i_{eq} = (1.15)(1.07) = 1.231$

Annual property tax = .0085X
FW of property tax = .0085X(F/A, 23.1%, 6) = .0909X

Adjusted return = 48,000 - .15(48,000 - X) - .0909X

Also = $X(1.231)^6 = 3.48X$

Therefore 40,800 + .15 X - .0909X = 3.48X
X = $11,927 purchase price

15-20

A solar system costs $6500 initially and qualifies for a federal tax credit (40% of cost, not to exceed $4,000). The cost of money is 10%, and inflation is expected to be 7% during the life of the system. The expected life of the system is 15 years with zero salvage value. The homeowner is in the 40% income tax bracket. The initial fuel saving is estimated at $500 for the first year and will increase in response to inflation. The annual maintenance cost of the system is established at 5% of the annualized cost of the system. What is the time required for the payback condition to be reached for this investment?

Solution

Adjust initial cost by tax credit: P = .60(6,500) = 3,900

Annualized cost: A = 3,900(A/P, 10%, 15) = 512.85

$1 + i_c = \frac{1.10(1+.40(.10))}{(1.05)} = 1.0895$ $1 + i_m = 1.05$ represents maintenance charge as a rate

PW of costs = 512.85(P/A, 8.95%, 15) = 512.85(8.086) = 4,146.67

$1 + i_{eq} = (1 + i)/(1 + i_f) = 1.10/1.07 = 1.028$

Try 9 years: PW = 500 (P/A, 2.8%, n) = 500 (7.868) = $3,934.18
Try 10 years: PW = 500 (P/A, 2.8%, n) = 500 (8.618) = $4,308.97

9 years < Payback < 10 years By interpolation payback = 9.6 years

15-21
The net cost of a solar system for a home is $8,000 and it is expected to last 20 years. If the value of money is 10%, inflation is expected to be 8%, and the initial annual fuel saving is $750, what is the time for the payback condition to be reached for the system? Assume the homeowner is in the 30% income tax bracket.

Solution

Annualize P: A = 8,000 (A/P, 10%, 20) = 940

$1 + i_c = (1.10)(1 + .10 \times .30) = 1.133$

PW of Cost = 940(P/A, 13.3%, 20) = 940(6.900) = 6,486

$1 + i_{eq} = (1 + i)/(1 + i_f) = 1.10/1.08 = 1.0185$

Try 9 years: PW = 940(P/A, 1.85%, n) = $6,171
Try 10 years: PW = 940(P/A, 1.85%, n) = $6,790

9 years < Payback < 10 years By interpolation payback = 9.5 years

15-22
An undeveloped percent of land in Gibson County, Tennessee was purchased in 1980 for $4,850. The property tax was $8 for the first year and is assumed to have increased by $2 per year each year thereafter. The capital gain tax is 13.6% of long-term capital gain. Inflation for the period is an 8% annual rate. A 16% rate of return on the investment is desired. What is the required sale price in 1985?

Solution

$1 + i_{eq} = (1.16)(1.08) = 1.2528$

FW of property tax = [8 + 2(A/G, 25.28%, 5)] [F/A, 25.28%, 5]
= [8+ 2(3.12)] [8.252]
= 91.74

Let X = selling price

Long-term capital gains tax = 0.136(X - 4,850) = .136X - 659.60

Adjusted return = X - [.136X - 659.60 + 91.74] = .864X + 567.86

Also = 4,850(1.2528) = 14,967.54

.864X + 567.86 = 14,967.54
.864X = 14,399.68
X = $16,666.31 selling price

15-23
The apparent interest rate is 9.18% and the real interest rate is 6%. The inflation rate is

a. 3.00%
b. 3.18%
c. 5.30%
d. 6.00%

Solution

i = i' + f + i'f
.0918 = .06 + f + .06f
.0318 = 1.06f
f = .03

The answer is a.

15-24
An investor is considering the purchase of a bond. The bond has a face value of $1,000, an interest rate of 6%, pays interest once a year, and matures in 8 years. This investor's real MARR is 25%. If the investor expects an inflation rate of 4% per year for the next 8 years, how much should he be willing to pay for the bond?

a. $250.50
b. $367.50
c. $384.74
d. $1,000.00

Solution

In order to earn a real 25% return with inflation of 4%, the nominal MARR must be equal to (1.25)(1.04) - 1 = 30%,

NPV = 0 at IRR
0 = -F.C. + 60(P/A, 30%, 8) + 1,000(P/F, 30%, 8)
F.C. = $250.50

The answer is a.

15-25
An investment in undeveloped land of $9,000 was held for four years and sold for $21,250. During this time property tax was paid that was, on the average, 0.4% of the purchase price. Inflation in this time period averaged 7% and the income tax was 15.2% of the long-term capital gain. What rate of return was obtained on the investment?

Solution

Long-term capital gains tax = 0.152(21,250 - 9,000) = $1,862

Property tax = .004 × 9,000 = \$36/year

FW of property tax = 36(F/A, i_{eq}, 4)

$1 + i_{eq} = \left(\frac{21{,}250 - 1{,}862}{9000}\right)^{\frac{1}{4}} = 1.2115$ (1^{st} estimate)

FW of property tax = 36(F/A, 21.15%, 4) = 36(5.47) = \$197

$1 + i_{eq} = \left(\frac{21{,}250 - 1{,}862 - 197}{9000}\right)^{\frac{1}{4}} = 1.2084$ (2^{nd} estimate)

Rate of return = $\left(\frac{1.2084}{1.07}\right)$ -1 = 12.9%

15-26

A company has designed a VLSI circuit and a production system to manufacture it. It is believed that it can sell 100,00 circuits per year if the price in then-current dollars is cut 20% per year (for example, if the unit price in the first year is \$100, then the price in years 2 through 5 would be \$80, \$64, \$51.20, and \$40.96). The required revenue for the five years is \$2,500.00 per year in today's dollars. The real and monetary costs of capital are 8.8% and 16.416%, respectively. What should the then-current dollar selling price be in each of the years 1 through 5?

Solution

Let R be the required revenue in year 1, then the required revenue in years 2 through 5 is .8R, .64R, .512R, and .0496R. Since these are in then-current \$,

$$(2{,}500{,}000)(P/A, 8.8\%, 5) = R(1.16416)^{-1} + 0.8R(1.16416)^{-2} + .064R(1.16416)^{-3} + 0.512R(1.16416)^{-4} + 0.4096R(1.16416)^{-5}$$

$$9{,}774{,}800 = 2.32523R$$

R = 4,203,804 or a unit price of

\$42.04 in year 1
\$33.63 in year 2
\$26.90 in year 3
\$21.52 in year 4
\$17.22 in year 5

15-27

An electronic device cost \$1,250 in 2001. If inflation has averaged 2% each year, the price of the device in 2008 is closet to

a. \$1,400
b. \$1,408
c. \$1,425
d. \$1,436

Solution

$F = (1 + f)^n$
$F = (1 + .02)^7$
$= \$1,435.86$

The answer is d.

15-28
A bond that pays no interest is called a zero-coupon bond. A $10,000 zero-coupon bond that matures in ten years can be purchased today. If the expected annual rate of inflation is 3% and the buyer's unadjusted MARR is 8%, what is the maximum that should be paid for the bond?

Solution

$i = i' + f + i'f$
$= .08 + .03 + .08(.03)$
$= .1124$

$P = 10,000(1 + .1124)^{-10}$
$= \$3,446.59$

15-29
Sylvia B. bought an 8% tax-free municipal bond. The cost of the bond was $10,000 and it will pay $800 each year for 20 years. The bond will mature at that time and return the original $10,000. If inflation is expected to average 3% during the period, what is the inflation adjusted rate of return?

a. 2.40%
b. 4.85%
c. 8.00%
d. 11.24%

Solution

$i = 8\%$ $f = 3\%$

$i = i' + f + i'f$
$.08 = i' + .03 + i'(.03)$
$i' = .0485 = 4.85\%$

The answer is b.

15-30
A vacant lot is purchased for $20,000. After five years the lot is to be offered for sale. If the buyer requires a before tax return on investments of 15% and inflation has averaged 4% per year over the five-year period the required selling price is nearest to

a. $30,650
b. $31,500
c. $48,950

d. $62,750

Solution

F = 20,000(F/P, 4%, 5)(F/P, 15%, 5)
= $48,947.74

The answer is c.