Exercise 9.10.1

Alice is violating the standard monotonicity assumption that she always prefers more eggs over less eggs.

Exercise 9.10.2

(a) 3;

(b) $2af$;

(c) $\frac{1}{f\sqrt{a}+2a}$.

Exercise 9.10.3

$\nabla u\,(f,a) = (2af, a^2)$, the equation of the tangent plane at $(A,F)^T$ is $3A^2F = a\,2AF + fA^2$, where (a,f) denotes a point on the plane (notice that the tangent plane is in fact a line).

Exercise 9.10.4

We need to solve for points (a, f) satisfying $\lambda\,(2af, a^2) = (f^2, 2af)$, which gives us the following equations: $2\lambda a = f$, $\lambda a = 2f$. This gives the solution $f = 0$ and either $\lambda = 0$ or $a = 0$. At these points, which are common to indifference curves of both agents, the tangent planes to these indifference curves are also common. This means that the indifference curves touch at these points.

Exercise 9.10.5

$\pi(w) = r(w) - c(w)$, so $\pi'(w) = 0$ is the same as $r'(w) = c'(w)$. Assuming revenue to be concave and cost to be convex, this is the condition for the maximum of the profit function. At the minimum of $\pi(.)$ it will then be $r'(w) < c'(w)$, and the minimum will be achieved either at $w = 0$ or at $w = w_{\max}$.

Exercise 9.10.6

Adam's indifference curve in the (f, p) space is $(A - pf)\, f^2 = c$ where c is a constant. To obtain his demand we have to maximize his utility as a function of f, at a given price p. That is, $2Af - 3pf^2 = 0$, yielding the demand curve $f = \frac{2A}{3p}$.

Exercise 9.10.7

Bob's demand is found by $\max a^2 f$ given that $pf + qa \leq M$. At the maximum, this constraint has to hold with equality, thus we have to compute $\max \frac{a^2}{p}(M - qa)$. This gives us Bob's demand for apples $a = \frac{2M}{3q}$. Using the relation $pf + qa = M$ we then get his demand fot fig leaves $f = \frac{M}{3p}$. N copies of of Bob will demand N times as much apples and fig leaves at given prices.

Exercise 9.10.8

If Alice fixes prices p and q then her revenue is $N\left(p\frac{M}{3p}+q\frac{2M}{3q}\right)=NM$. Suppose her unit cost of producing fig leaves is 0 and the cost of producing an apple is $c>0$. Then her profit if prices are p and q is $\pi(p,q)=NM-cN\frac{2M}{3q}$. This implies that she gets the highest profit when q is infinite, and the demand for apples is 0.

Exercise 9.10.9

When fig leaves and apples are perfect substitutes, Adam's demand is given by

$$f = \begin{cases} \frac{A}{p}; \text{ if } p < \frac{1}{2} \\ 0; \text{ if } p > \frac{1}{2} \\ \text{any point in } \left[0, \frac{A}{2}\right]; \text{ if } p = \frac{1}{2} \end{cases}.$$

In case of perfect complements, notice that in order to maximize his utility, Adam will have to have $f = 2a$. Thus, his demand will be given by $2(A - pf) = f$, yielding $f = \frac{2A}{1+2p}$ for his demand curve.

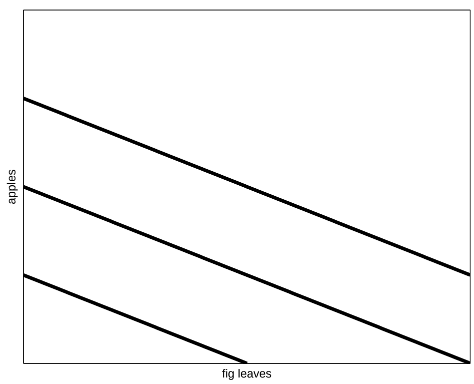


Figure 9.10.9(a)

Figure 9.10.9(b)

Exercise 9.10.10

(a) See figure 9.10.10.

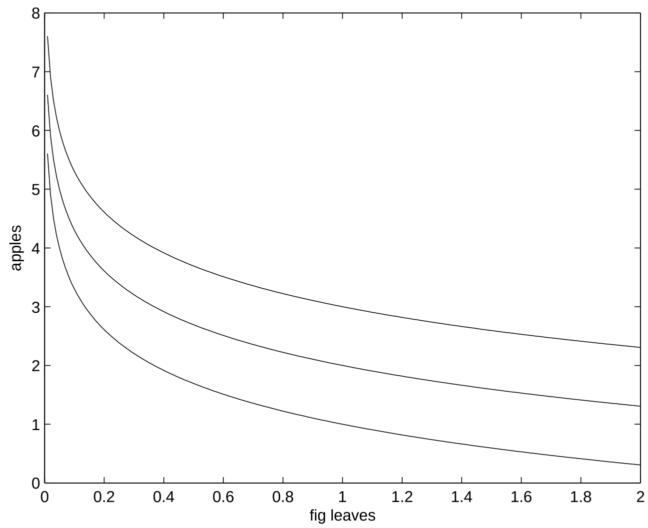


Figure 9.10.10

(b) $f = \frac{1}{p}$.

(c) If adam started with F and ended up with f fig leaves then his utility gain is the integral under his demand curve, from the initial price $p = \frac{1}{F}$ to the end price $p = \frac{1}{f}$, that is

$$\Delta u = \int_{\frac{1}{F}}^{\frac{1}{f}} \frac{1}{p} dp = \ln F - \ln f$$

When $F = 0$ his initial utility is $-\infty$ (since $\ln 0 = -\infty$), thus the change in his utility will have to be infinite, so it is clear that the above integral will be divergent (meaning that it is equal to $\pm\infty$). Indeed, it is easy to check that it is the case.

Exercise 9.10.11

To find the contract curve we first find the set of efficient trades by solving for points where $\nabla u(f, a) = \lambda \nabla u(F - f, A - a)$. In case (a) this gives us two equations $(2af, f^2) = \lambda\left(-2(A - a)(F - f), -(F - f)^2\right)$, so if we divide them, we obtain the equality $\frac{a}{f} = \frac{(A-a)}{(F-f)}$ (notice that this is the equality of marginal rates of substitution). We can rearrange this to get the set of all efficient outcomes: $a = A\frac{f}{F}$, $f \in [0, F]$. Since the utilities of the agents are 0 at their initial endowment, this is also the contract curve. At the Walrasian equilibrium we have that the price vector $(p, 1)$ also points into the same direction as the utility gradients of both agents, which means that we get an additional equation (again by dividing to get rid of the multiplyer), $\frac{p}{1} = \frac{a}{f}$. Along with the budget constraint (take for instance Adam's), $pf = A - a$, this gives us sufficiently many equations to compute the Walrasian equilibrium - we can solve for p in terms of a and f, then plug in the expression for a, and finally nail down the solution for f with the remaining equation. The Walrasian equilibrium is $f = \frac{F}{2}$, $a = \frac{A}{2}$, and $p = \frac{A}{F}$. For case (b) we can proceed in the same way, to obtain the contract curve $a = \frac{f(A+2)-F+A}{F+2}$, where we now have to be careful with the interval of possible values for f, which are determined from the condition that each agent get at least as much utility as his initial endowment gives him.

Exercise 9.10.12

Figure 9.10.12 illustrates Adam's indifference curves.

The math is as follows. If Eve can commit to not sell more than s fig leaves, at a fixed price p, then Adam is solving the following maximization problem

$$\max_{f \leq s} A - pf + \ln f.$$

Take derivative to obtain that at an interior maximum, $f = \frac{1}{p}$. Thus Adam's demand is $f = \begin{cases} \frac{1}{p}; \text{ if } \frac{1}{p} < s \\ s; \text{ if } \frac{1}{p} \geq s \end{cases}$. The number of apples he will be left with is $a = A - pf = \begin{cases} A-1; \text{ if } \frac{1}{p} < s \\ A - sp; \text{ if } \frac{1}{p} \geq s \end{cases}$. This implies that the monopoly locus is made out of the two perpendicular lines in figure 9.10.12, and the monopoly point M is at the point $(0, A-1)$.

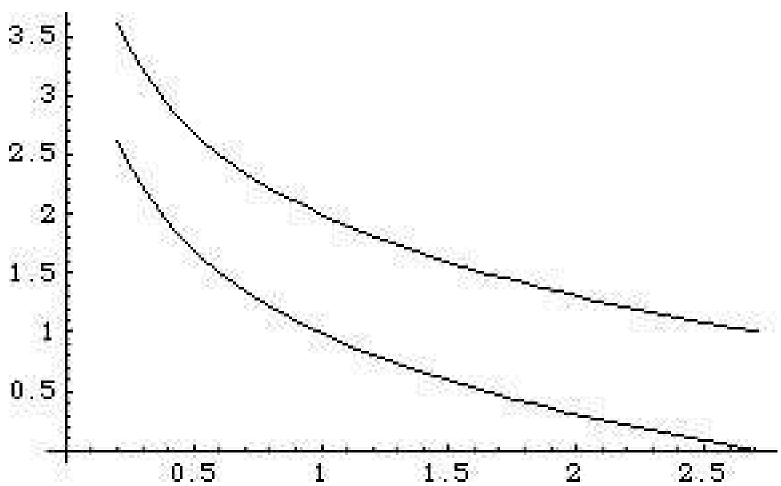


Figure 9.10.12

Exercise 9.10.13

$u(f,a) = \min\{f,a\}$.

(a) See the solution to 9.10.9.

(b) The demand is obtained as the intersection of the budget line $a + pf = A + pF$. and the line on which the indifference curves have the kink, $a = \frac{f}{2}$. Thus, the demand is $f = \frac{A+pF}{p+\frac{1}{2}}$.

(c) If we integrate the area under the demand curve from the price at the initial endowment, $p = \infty$, to some price p_0 we obtain

$$\int_{p_0}^{\infty} \left(\frac{A+pF}{p+\frac{1}{2}}\right) dp = \int_{p_0}^{\infty} \left(F + \frac{A-\frac{F}{2}}{p+\frac{1}{2}}\right) dp = \left(Fp - \left(A - \frac{F}{2}\right)\ln\left(p + \frac{1}{2}\right)\right)_{p_0}^{\infty} = \infty$$

Obviously, the change in Adam's utility is not infinite - notice that his utility isn't quasi-linear. For the case where $u(f,a) = f + 2a$ the analysis is analogous to the analysis of exercise 9.10.10 since his utility function is then again quasi-linear, except that there are no problems when $F = 0$.

Exercise 9.10.14

First compute Adam's demand: $\max_f (A - p(f - F))^2 f$, which gives the condition $(A - p(f - F))(A - p(3f - F)) = 0$. One root of this equation is a local minimum of Adam's utility, and the other one gives the demand $f = \frac{1}{3}\left(\frac{A}{p} + F\right)$. A perfectly discriminating monopolist charges the lowest price $p = 1$, which is obtained when $f = \frac{A+F}{3}$. So the area under the demand function is

$$\int_F^{\frac{A+F}{3}} \frac{3f - F}{A} df = \frac{1}{6A}\left(A^2 - 4F^2\right).$$

On the other hand, the monopolist trade point is the intersection of Adam's indifference curve $a^2 f = A^2 F$ with the contract curve $2f = a$ (it is immediate to compute that this is the contract curve). That is, $f = \sqrt[3]{\frac{A^2 F}{4}}$ and $a = \sqrt[3]{2A^2 F}$, and since Alice's profit is $\pi = A - a - (f - F)$ the above integral is clearly not equal to π.

Exercise 9.10.15

(a) See figure 9.10.15 below.

(b) No.

(c) Auction off the shovels.

(d) See figure 9.10.15 below.

(e) It isn't a good measure since in the case of health insurance the willingness to pay may sometimes be constrained by the budget, rather than the seriousness of the illness.

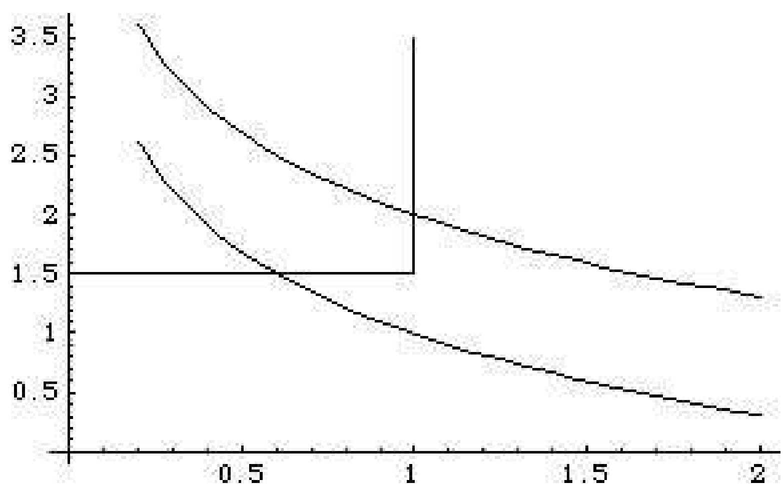


Figure 9.10.15

Exercise 9.10.16

Since the pumping stations know that the next day the price will be higher, they want to keep the supply down in order to hike up the price already on the given day. Another way to put this is that since the price will go up the next day, their opportunity cost of sales on the given day is increased, so the supply curve shifts upward.

0.1 Exercise 9.10.17

See figure 9.10.17 below. If the supply intersects the demand on a horizontal segment, then this means that the number of buyers will be lower than the number of owners willing to sell at that price, meaning that some owners will have to be rationed. If the demand cuts the supply in the vertical segment, then fn cars will be sold in equilibrium.

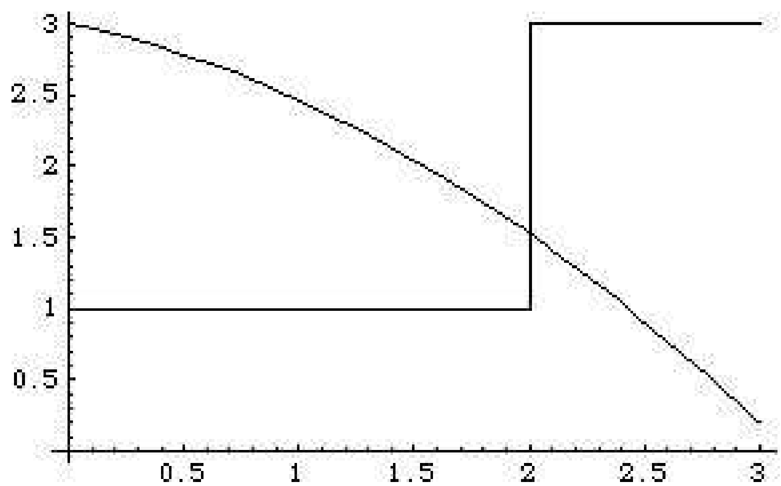


Figure 9.10.17

Exercise 9.10.18

(a) This is very simple and follows directly from the definition of expectation.

(b) Since the dealers can't observe the quality of the cars beforehand, the owners will sell all cars at price p. Because each dealer is risk-neutral, he will only buy a car if his expected resale price is higher than the price he had bought the car for, that is $fL + (1-f)\,P - p > 0$. After rearranging, we obtain the desired inequality. This means that then in fact all dealers will buy the cars, hence indeed the fraction of lemons sold will equal f. The demand is made out of two straight line segments, see figure 9.10.18(a) below. If $f > \frac{P-p}{P-L}$, then in the long run, all the dealers will be making negative profit, so that they will be dropping out of the market. If we assume that the lemons will be sold first then this implies that the fraction of lemons sold will be increasing, thus making even more dealers to drop out of the market.

(c) If a dealer believes only lemons will be sold, then he will only sell a car if $L > p$ which is not true. Hence, no dealer will buy a car and indeed only the lemons will be offered, but not sold, so the dealers' expectations are always rational in that case. See figure 9.10.18(b) below.

(d) This follows directly from parts (b) and (c).

Figure 9.10.18(a)

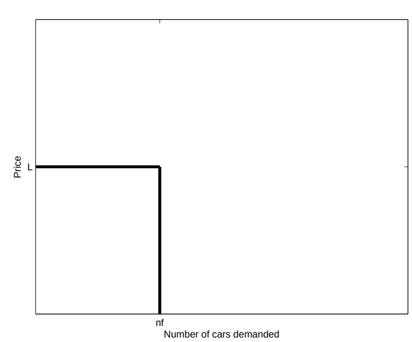
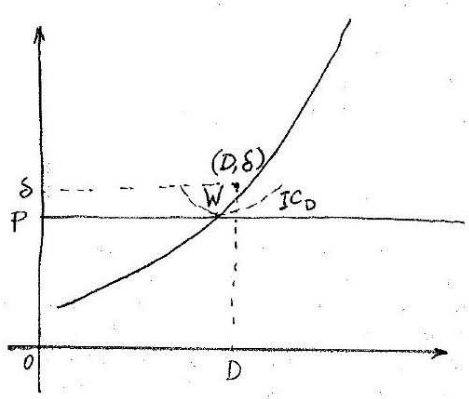
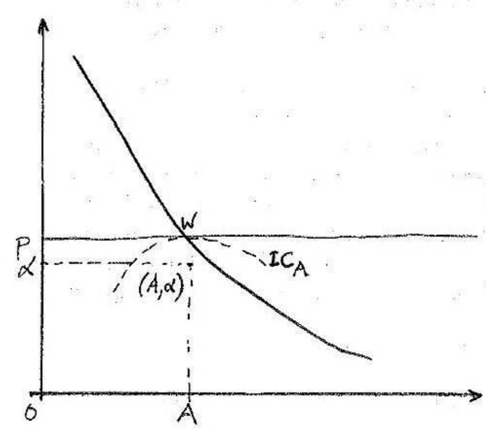


Figure 9.10.18(b)

Exercise 9.10.19

If the supply of N Dollys is given by $w = w(p)$, then the supply of 1 Dolly is given by $w = \frac{1}{N}w(p)$, which means that in the quantity-price space, the graph of Dolly's supply is obtained by "shrinking" the graph of $w(p)$ by factor $\frac{1}{N}$ toward the p-axis. Similarly for the demand. See fig9.10.19 for the picture. Assume that there is another point where Alice gets (A, α) and Dolly gets (D, δ). The only way that such improvement is possible is if the supply is at least as big as the demand, and the money demanded is at most as big as the money received. Hence, $MD \geq NA$ and $MD\delta \leq NA\alpha$. The first inequality implies $\frac{1}{MD} \leq \frac{1}{NA}$, multiplying this and the second inequality we obtain $\delta \leq \alpha$. At the Walrasian allocation it is also true that $MD_W = NA_W$, and $MD\delta_W = NA\alpha_W$, so that $\delta_W = \alpha_W$. On the other hand, since A_W is Alice's optimal choice at price α_W, it must be that A is not available at price α_W, so that $A\alpha_W > A_W\alpha_W$, implying that $\alpha < \alpha_W$. Similarly we obtain that $\delta > \delta_W$, so we get that any Pareto-improvement would need to have $\alpha < \delta$.





Exercise 9.10.20

Let (A_W, α_W) and (D_W, δ_W) be the Walrasian allocation, and let (A_0, α_0) and (D_0, δ_0) be the initial allocation of wool and money. Let p be the Walrasian price of wool (price of money is normalized at 1), and suppose there existed Pareto-improving bundles (A, α) and (D, δ), and suppose that (A, α) is a strict improvement for Alice. Then it must be that at price p, (A, α) was unaffordable to Alice, so that $Ap+\alpha > A_W p+\alpha_W$. Similarly, for Dolly, the bundle (D, δ) must have been weakly unaffordable, $Dp+\delta \geq D_W p + \delta_W$. This implies that $(D + A)p + \alpha + \delta > (D_W + A_W)p + \alpha_W + \delta_W$. By assumption, (A, α) and (D, δ) are feasible, so that $A + D = A_0 + D_0$ and $\alpha + \delta = \alpha_0 + \delta_0$. Since Walrasian allocation is feasible, we also have $D_W + A_W = A_0 + D_0$ and $\alpha_W + \delta_W = \alpha_0 + \delta_0$, hence, $(A + D)p + \alpha + \delta = (D_W + A_W)p + \alpha_W + \delta_W$, which is a contradiction.

Exercise 9.10.21

The following reserve prices will work. On the supply side: $p_i^S = i$, and on the demand side $p_j^D = j$ for $i = 1, ..., 10$ and $j = 1, ..., 10$. Then any price $p \in [5, 6]$ will be a Walrasian equilibrium price because at any such price there will be precisely 5 sellers willing to sell and 5 buyers willing to buy. Suppose the buyers tell the truth about their reserve prices. Then the Walrasian price will be set at $p = 6$. So all the buyers with reserve prices strictly more than 6 will be happier with telling the truth and trading than lieing and not trading. The buyer with reserve price exactly equal to 6 is indifferent between telling the truth or not, since his net gain is 0 in any case. So all the buyers are playing a best response. Clearly, the sellers are playing a best reply strategy since they do not really move anyway. So this is a Nash equilibrium.

Exercise 9.10.22

To maximize the utility observe that $x, t \geq 0$, and $u(x,t)$ is decreasing in x for every t and decreasing in t for every x. Hence the unique maximum of the utility is at $x = t = 0$. It is clear that for every (X_1, T_1) we can find an $X_2 < X_1$ and a $T_2 > T_1$ such that $u(X_2, T_2) > u(X_1, T_1)$. In particular, for every T the limit of utility when pain goes to 0 is 0, which is the maximal possible utility. Also see figure 9.10.22 below.

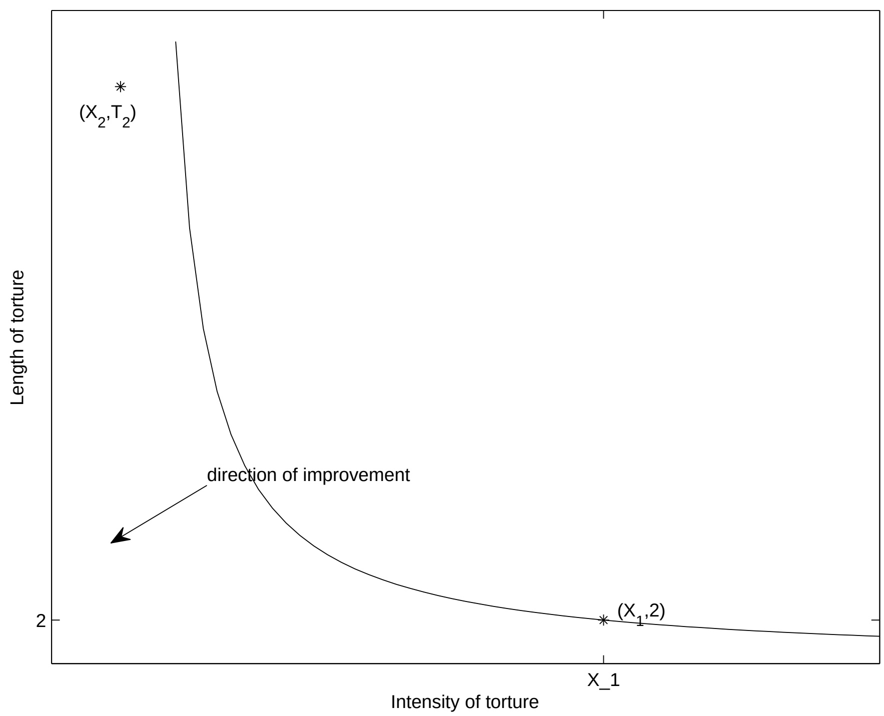


Figure 9.10.22