Exercise 5.9.1

See figure 5.9.1

	d_9	d_7	d_5	d_3	d_1
d_{10}	1.0 0.0	1.0 0.0	1.0 0.0	1.0 0.0	1.0 0.0
d_8	0.19 0.81	0.64 0.36	0.64 0.36	0.64 0.36	0.64 0.36
d_6	0.19 0.81	0.51 0.49	0.36 0.64	0.36 0.64	0.36 0.64
d_4	0.19 0.81	0.51 0.49	0.75 0.25	0.16 0.84	0.16 0.84
d_2	0.19 0.81	0.51 0.49	0.75 0.25	0.81 0.09	0.04 0.96
d_0	0.19 0.81	0.51 0.49	0.75 0.25	0.81 0.09	0.99 0.01

Figure 5.9.1

Exercise 5.9.2

The successive elimination proceeds as follows. Step 1: $d6 >> d10$ for player I. Step 2: $d7 >> d9$ for player II. Step 3: $d6 >> d8$ and $d6 \geq d0$ for I. Step 4: $d7 >> d1$ for II. Step 5: $d6 \geq d2$ for I. Step 6: $d5 \geq d3$ for II. Step 7: $d6 \geq d4$ for I. Step 8: $d7 \geq d5$ for II. The resulting outcome is the subgame perfect equilibrium. Whenever successive elimination of dominated strategies results in a single outcome, that outcome must be a Nash equilibrium. In this game, Nash equilibrium is unique, thus it must coincide with the subgame perfect equilibrium.

Exercise 5.9.3

(a) This can properly be called a zero-sum game. The Von Neumann and Morgenstern utility assigned to the outcomes where Jerry hides and Tom finds him are 1 for Tom and −1 for Jerry. All other outcomes have a utility of 1 for Jerry and −1 for Tom.

(b) See figure 5.9.3(a).

Tom

Jerry	BBB	BBD	BBK	BDB	BDD	BDK	BKB	BKD	BKK	DBB	DBD	DBK	DDB	DDD
B	−1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1
D	1	1	1	−1	−1	−1	1	1	1	1	1	1	−1	−1
K	1	1	−1	1	1	−1	1	1	−1	1	1	−1	1	1

Jerry	DDK	DKB	DKD	DKK	KBB	KBD	KBK	KDB	KDD	KDK	KKB	KKD	KKK
B	1	1	1	1	1	1	1	1	1	1	1	1	1
D	−1	1	1	1	1	1	1	−1	−1	−1	1	1	1
K	−1	1	1	−1	1	1	−1	1	1	−1	1	1	−1

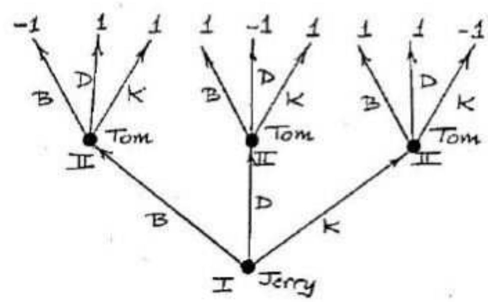
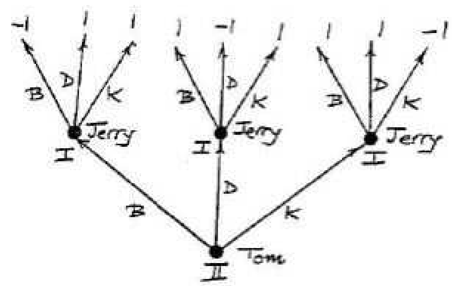


Figure 5.9.3(a)

(c) See figure 5.9.3(b).

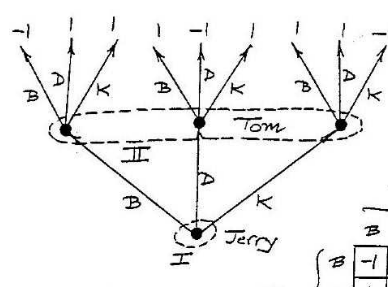
(d) See figure 5.9.3(c).

(e) $\{(B, BDK), (D, BDK), (K, BDK)\}$ is the set of pure strategy Nash equilibria in part (b). In part (c), the pure strategy Nash equilibria are of the form (w, xyz) where $w \in \{B, D, K\}$. $x \neq B$, $y \neq D$, and $z \neq K$. There are no pure strategy Nash equilibria in part (d).



Jerry \ Tom	B	D	K
BBB	-1	1	1
BBD	-1	1	1
BBK	-1	1	-1
BDB	-1	-1	1
BDD	-1	-1	1
BDK	-1	-1	-1
BKB	-1	1	1
BKD	-1	1	1
BKK	-1	1	-1
DBB	1	1	1
DBD	1	1	1
DBK	1	1	-1
DDB	1	-1	1
DDD	1	-1	1
DDK	1	-1	-1
DKB	1	1	1
DKD	1	1	1
DKK	1	1	-1
KBB	1	1	1
KBD	1	1	1
KBK	1	1	-1
KDB	1	-1	1
KDD	1	-1	1
KDK	1	-1	-1
KKB	1	1	1
KKD	1	1	1
KKK	1	1	-1

Figure 5.9.3(b)



Jerry \ Tom	B	D	K
B	-1	1	1
D	1	-1	1
K	1	1	-1

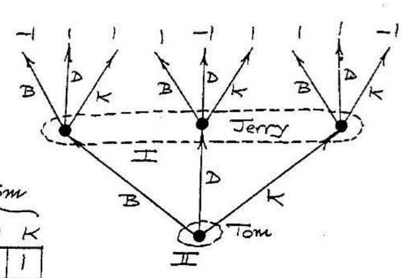


Figure 5.9.3(c)

Exercise 5.9.4

$$A^T = \begin{bmatrix} 2 & -1 \\ 1 & 4 \\ 3 & 0 \end{bmatrix} \; B^T = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & 0 \end{bmatrix} \text{ and } C^T = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 2 & 4 \end{bmatrix}.$$

Exercise 5.9.5

The payoff matrices for the left bimatrix game are:

$$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}.$$

The payoff matrices for the right bimatrix game are:

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}.$$

The left bimatrix game is symmetric and the right one is not. The payoff matrices for both players are not symmetric in the left bimatrix game, and are symmetric for both players in the right bimatrix game.

Exercise 5.9.6

(a) $z = (2, 3) \in A \cap B \cap C$.

(b) $z = (2, 2) \in A \cap B$.

(c) $z = (1, 2) \in A$.

(d) $z = (2, 1) \notin A \cup B \cup C$.

Exercise 5.9.7

Six times

Exercise 5.9.8

Figure 5.9.8 below has the extensive form, which in strategic form is

	L	R
l	0, 0	1, 0
r	1, 1	1, 1

Elimination of weakly dominated strategies eliminates (l, R), which is a subgame-perfect equilibrium of the game in extensive form.

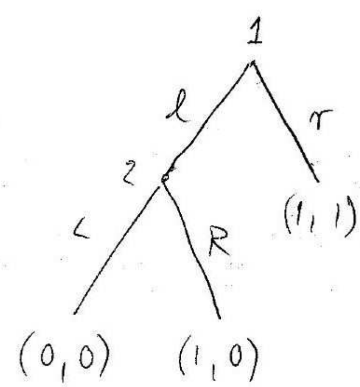


Figure 5.9.8

Exercise 5.9.9

With the strategy pair (ADD, AAD), player I gets 0 with probability $\frac{1}{6}$ and, with probability $\frac{1}{6}$, he moves on to a lottery in which he gets 0 with probability $\frac{1}{5}$ and a with probability $\frac{4}{5}$. If the game gets this far it will end with certainty. Therefore, $\pi_1(ADD, AAD) = \frac{1}{6}0 + \frac{5}{6}(\frac{1}{5}1 + \frac{4}{5}a) = \frac{1}{6} + \frac{2}{3}a$. Similar reasoning shows that $\pi_2(ADD, AAD) = \frac{1}{6}1 + \frac{5}{6}(\frac{1}{5}0 + \frac{4}{5}1) = \frac{5}{6}$.

Exercise 5.9.10

See the table below.

	uU	uD	dU	dD
uU	$\frac{3}{2}$, $\frac{3}{2}$	$\frac{3}{2}$, $\frac{3}{2}$	1, $\frac{5}{2}$	1, $\frac{5}{2}$
uD	$\frac{3}{2}$, $\frac{3}{2}$	$\frac{3}{2}$, $\frac{3}{2}$	1, $\frac{5}{2}$	1, $\frac{5}{2}$
dU	$\frac{5}{2}$, 1	$\frac{5}{2}$, 1	$\frac{5}{2}$, $\frac{5}{2}$	5, $\frac{1}{2}$
dD	$\frac{5}{2}$, 1	$\frac{5}{2}$, 1	$\frac{1}{2}$, 5	4, 4

Exercise 5.9.11

If Colonel Blotto knew what a dominated strategy was, he would put his five companies at the five most valuable locations.

Exercise 5.9.12

Now the subgame perfect equilibrium is Bob playing 4 and Alice playing 4, resulting in the outcome $(16, 16)$.

Exercise 5.9.13

(a) If Alice precommits before Bob, then Bob will have to best-reply to Alice's precommitment which is the same as if they play the Stackelber game with Bob being the follower.

(b) The argument is symmetric to that in (a).

(c) If they precommit simultaneously, it is the same as if they simultaneously chose their production levels.

Exercise 5.9.14

If Alice and Bob each precommit to a strategy then the outcome is determined. This explains the upper left-hand 2×2 part of the payoff table. If Bob precommits to a strategy but Alice doesn't then Alice will choose her optimal outcome among her two remaining possibilities. This outcome is then written into the payoff table. Successive elimination of dominated strategies proceeds like this: in step 1 $\Box$ weakly dominates 6 and 4 for Alice; in step 2 4 strongly dominates $\Box$ and 3 for Bob. Resulting outcome is $(16, 16)$.

Exercise 5.9.15

(a) First Bob decides whether to enter the market or stay out. Then, after observing Bob's decision, Alice either fights or acquiesces. This gives the extensive form in Figure 5.17(a). To see that $(in, acquiesce)$ is the subgame perfect equilibrium proceed by backward induction. If Bob enters then Alice will optimally acquiesce. Comparing this to not entering, Bob will do best by entering.

(b) The strategies are *in*, *out* for Bob and *acquiesce*, *fight* for Alice. If Bob plays *out* then their payoffs are independent of what Alice does. Nash equilibria are (*fight*, *out*) and (*acquiesce*, *in*). The former does not survive under successive elimination of dominated strategies.

(c) Alice's threat is not credible because if Bob enters, then it is for her optimal to not fight.

Exercise 5.9.16

(a) See figure 5.9.16.

(b) The subgame perfect equilibrium is (*commit_acquiesce,out_in*).

(c) By expanding her production and thus sinking a cost and making it cheaper for herself to flood the market (see also exercise 5.19.17).

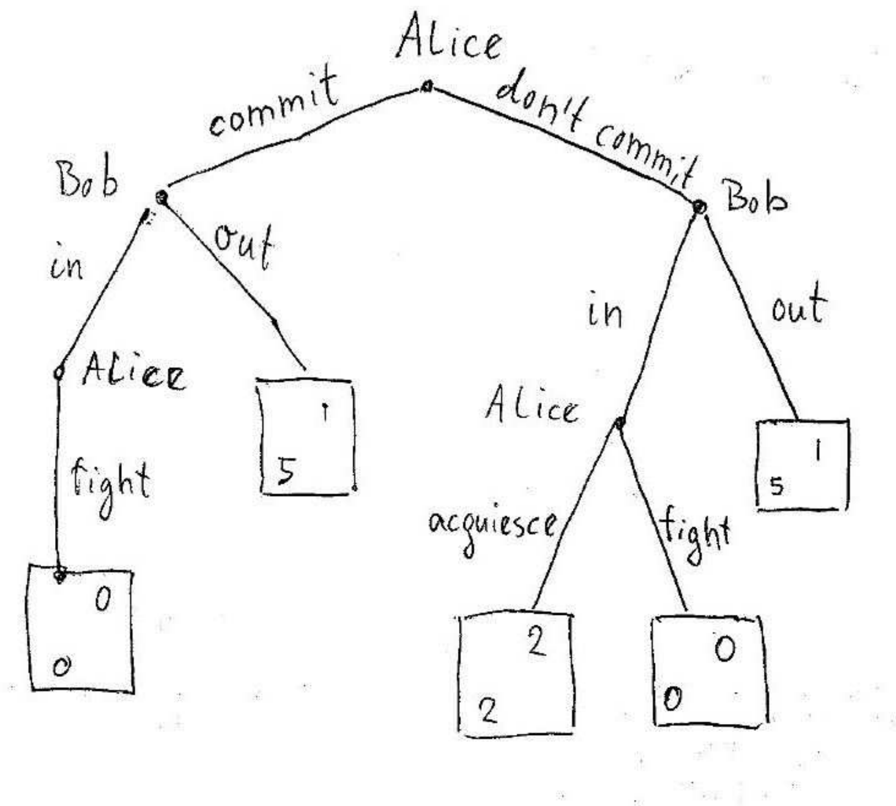


Figure 5.9.16

Exercise 5.9.17

(a) See figure 5.9.17.

(b) The subgame perfect equilibrium is (*increase_fight_acquiesce, out_in*).

(c) Investing in extra capacity is rational because it makes Alice's threat credible, thus raising her equilibrium payoff from 2 to 3.

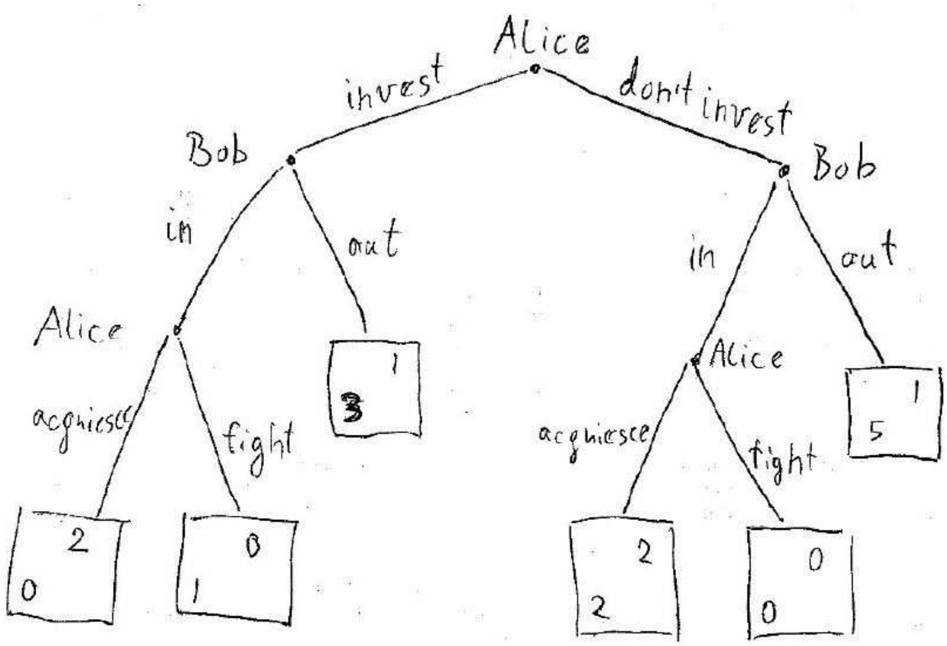


Figure 5.9.17

Exercise 5.9.18

If Alice invests it is for Bob best to slack and consume \$$3m$ instead of \$$2m$.

Exercise 5.9.19

(a) The extensive form is depicted in figure 5.9.19. At every node where Alice has to move after a previous entry it is for her optimal to acquiesce so that entry is optimal for Bob and for Chris.

(b) The outcome in the previous town does not affect the decision of the next entrant.

(c) In real life it seems more plausible that the outcome in the previous town does affect the decision of the next potential entrant, in particular, by fighting off Bob, Alice may show that she is a tough player. Thus, by fighting Alice is able to scare off her future competitors.

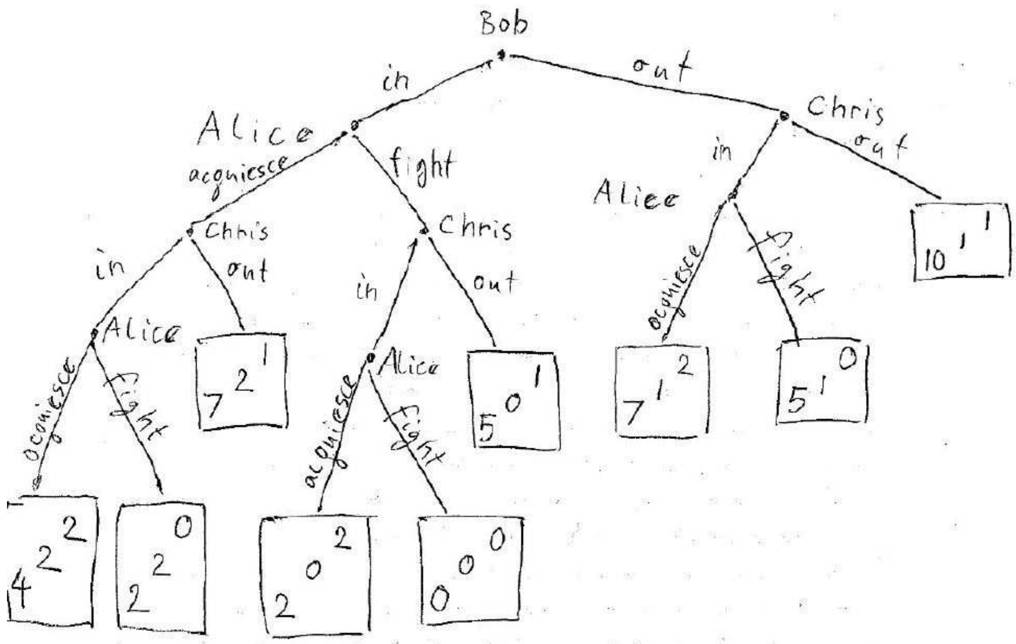


Figure 5.9.19

Exercise 5.9.20

(a) The subgame-perfect equilibrium is $(DDDDD, DDDDD)$. This would also be the result of successively deleting weakly dominated strategies.

(b) No. If the president of Yalebridge is offered $100000, he may believe he is playing an irrational player and move across.

Exercise 5.9.21

(a) The strategic form is

	L	H
L	999,999 999,999	999,997 1,000,001
H	1,000,001 999,997	1,000,000 1,000,000

This is a prisoner's dilemma, since both players would deviate from H, resulting in the Nash equilibrium (L, L).

(b) Step 1_a: $999,999$ weakly dominates $1,000,000$ for player I. Step 1_b: $999,999$ weakly dominates $1,000,000$ for player II. After these strategies have been deleted, the new payoff matrix is similar to the initial one, only that now $999,999$ is the highest possible strategy for each player. Thus, in step n_a $1,000,000 - n$ weakly dominated $1,000,000 - n + 1$ for player I, and in step n_b $1,000,000 - n$ weakly dominated $1,000,000 - n + 1$ for player II. After completing steps $999,000_a$ and $999,000_b$ we are left with only one strategy, 1000, for each player. This is the unique pure-strategy Nash equilibrium of the game.

(c) Now strategy $1,000,000$ is an approximate best reply to $1,000,000$. To see this notice that if I plays $1,000,000$ and II plays anything lower than $999,999$ then he gets less then or equal than if he played $1,000,000$. If he plays $999,999$ he gets \$1 more, but by the assumption $\varepsilon > 1$.

(d) No, because people are likely not to care about one dollar when such large amounts are at stake.

Exercise 5.9.22

If both players always play GRIM they obtain a payoff of $\frac{1}{n}n = 1$ each. If one of the players contemplates deviating from GRIM, then it he will quickly realize that it is for him best to deviate in the last period. Then his payoff is increased to $\frac{1}{n}(n-1+3) = 1 + \frac{2}{n}$. Thus, in order to any player not be tempted, we need $\varepsilon > \frac{2}{n}$, implying that $n > \frac{2}{\varepsilon}$.

Exercise 5.9.23

(a) One solution is the following. To make the game the simplest possible, assume that at the start of the game, any player holds at most \$1, that no player can own the bottle twice, and that there are 100 players. Also, assume that each player only considers the highest possible price, if there is one at all, when he is selling the bottle, and that player i can only sell the bottle to player $i+1$. If a player gets the bottle and sells it off, his payoff is 1, if he has the bottle but is unable to sell it, his payoff is -1, and if he never has the bottle, his payoff is 0. Thus at node player i decides whether to buy or not, if he doesn't buy, then the game ends with appropriate payoffs, if he buys, the game proceeds to the next node. If player 100 buys the bottle, the game also end. The subgame perfect equilibrium is that no player ever buys the bottle. Ths is easy to see using backward induction: at node 100 player 100 won't buy the bottle, hence player 99 won't buy it, and so on to player 1.

(b) If you are rational and believe that other people are also rational, you shouldn't buy the bottle.

Exercise 5.9.24

If Pandora is the only player, then being better informed means that she can base her decision on her estimate (average) over a smaller set of possible states, which means that her decision will be more precise. Thus, her payoff will be at least as big as if her decision is based on the average over a larger set (i.e. if Pandora is completely informed, then she can take the best outcome for that particular case). For an example where there are many players consider a Cournot versus a Stackelberg game of Exercise 5.9.14. In a Cournot game, the follower is better informed than in a Stackelberg game, yet this makes him worse off because the leader knows that the follower will be better informed.

Exercise 5.9.25

If the Alice does not know that Bob is better informed, then she will play the Nash equilibrium of the Cournot game, which means that Bob will be no worse off than if he weren't better informed. See also the second part of the answer to the previous exercise.