APPENDIX D

SOME USEFUL NETWORK THEOREMS

Introduction

In this appendix we review three network theorems that are useful in simplifying the analysis of electronic circuits: Thévenin’s theorem, Norton’s theorem, and the source-absorption theorem.

D.1 Thévenin’s Theorem

Thévenin’s theorem is used to represent a part of a network by a voltage source $V_t$ and a series impedance $Z_t$, as shown in Fig. D.1. Figure D.1(a) shows a network divided into two parts, A and B. In Fig. D.1(b), part A of the network has been replaced by its Thévenin equivalent: a voltage source $V_t$ and a series impedance $Z_t$. Figure D.1(c) illustrates how $V_t$ is to be determined: Simply open-circuit the two terminals of network A and measure (or calculate) the voltage that appears between these two terminals. To determine $Z_t$, we reduce all external (i.e., independent) sources in network A to zero by short-circuiting voltage sources and open-circuiting current sources. The impedance $Z_t$ will be equal to the input impedance of network A after this reduction has been performed, as illustrated in Fig. D.1(d).

D.2 Norton’s Theorem

Norton’s theorem is the dual of Thévenin’s theorem. It is used to represent a part of a network by a current source $I_n$ and a parallel impedance $Z_n$, as shown in Fig. D.2. Figure D.2(a) shows a network divided into two parts, A and B. In Fig. D.2(b), part A has been replaced by its Norton’s equivalent: a current source $I_n$ and a parallel impedance $Z_n$. The Norton’s current source $I_n$ can be measured (or calculated) as shown in Fig. D.2(c). The terminals of the network being reduced (network A) are shorted, and the current $I_n$ will be equal simply to the short-circuit current. To determine the impedance $Z_n$, we first reduce the external excitation in network A to zero: That is, we short-circuit independent voltage sources and open-circuit independent current sources. The impedance $Z_n$ will be equal to the input impedance of network A after this source-elimination process has taken place. Thus the Norton impedance $Z_n$ is equal to the Thévenin impedance $Z_t$. Finally, note that $I_n = V_t/Z$, where $Z = Z_n = Z_t$. 

©2015 Oxford University Press
Reprinting or distribution, electronically or otherwise, without the express written consent of Oxford University Press is prohibited.
**Example D.1**

Figure D.3(a) shows a bipolar junction transistor circuit. The transistor is a three-terminal device with the terminals labeled E (emitter), B (base), and C (collector). As shown, the base is connected to the dc power supply $V^+$ via the voltage divider composed of $R_1$ and $R_2$. The collector is connected to the dc supply $V^+$ through $R_3$ and to ground through $R_4$. To simplify the analysis, we wish to apply Thévenin’s theorem to reduce the circuit.

**Solution**

Thévenin’s theorem can be used at the base side to reduce the network composed of $V^+$, $R_1$, and $R_2$ to a dc voltage source $V_{bb}$.

\[
V_{bb} = V^+ \frac{R_3}{R_1 + R_2}
\]
D.3 Source-Absorption Theorem

Consider the situation shown in Fig. D.4. In the course of analyzing a network, we find a controlled current source $I_x$ appearing between two nodes whose voltage difference is the controlling voltage $V_x$. That is, $I_x = g_m V_x$, where $g_m$ is a conductance. We can replace this controlled source by an impedance $Z_x = V_x/I_x = 1/g_m$, as shown in Fig. D.4, because the current drawn by this impedance will be equal to the current of the controlled source that we have replaced.

Figure D.4 The source-absorption theorem.
Example D.2

Figure D.5(a) shows the small-signal, equivalent-circuit model of a transistor. We want to find the resistance $R_m$ “looking into” the emitter terminal E—that is, the resistance between the emitter and ground—with the base B and collector C grounded.

![Diagram of transistor model]

**Solution**

From Fig. D.5(a), we see that the voltage $v_x$ will be equal to $-v_e$. Thus, looking between E and ground, we see a resistance $r_x$ in parallel with a current source drawing a current $g_m v_e$ away from terminal E. The latter source can be replaced by a resistance $\left(\frac{1}{g_m}\right)$, resulting in the input resistance $R_{in}$ given by

$$R_{in} = r_x \parallel \left(\frac{1}{g_m}\right)$$

as illustrated in Fig. D.5(b).

**EXERCISES**

D.1 A source is measured and found to have a 10-V open-circuit voltage and to provide 1 mA into a short circuit. Calculate its Thévenin and Norton equivalent source parameters.

*Ans.* $V_t = 10$ V; $Z_t = Z_n = 10$ kΩ; $I_n = 1$ mA

D.2 In the circuit shown in Fig. ED.2, the diode has a voltage drop $V_D \simeq 0.7$ V. Use Thévenin’s theorem to simplify the circuit and hence calculate the diode current $I_D$.

*Ans.* 1 mA
The two-terminal device M in the circuit of Fig. ED.3 has a current $I_M \simeq 1$ mA independent of the voltage $V_M$ across it. Use Norton’s theorem to simplify the circuit and hence calculate the voltage $V_M$.

Ans. 5 V
D.4 Find the output voltage and output resistance of the circuit shown in Fig. PD.4 by considering a succession of Thévenin equivalent circuits.

D.5 Repeat Example D.2 with a resistance $R_B$ connected between $B$ and ground in Fig. D.5 (i.e., rather than directly grounding the base $B$ as indicated in Fig. D.5).

D.6 Figure PD.6(a) shows the circuit symbol of a device known as the $p$-channel junction field-effect transistor (JFET). As indicated, the JFET has three terminals. When the gate terminal $G$ is connected to the source terminal $S$, the two-terminal device shown in Fig. PD.6(b) is obtained. Its $i$–$v$ characteristic is given by

$$i = I_{DSS} \left[ \frac{v}{V_P} - \left( \frac{v}{V_P} \right)^2 \right] \quad \text{for } v \leq V_P$$

$$i = I_{DSS} \quad \text{for } v \geq V_P$$

where $I_{DSS}$ and $V_P$ are positive constants for the particular JFET. Now consider the circuit shown in Fig. PD.6(c) and let $V_P = 2$ V and $I_{DSS} = 2$ mA. For $V^+ = 10$ V show that the JFET is operating in the constant-current mode and find the voltage across it. What is the minimum value of $V^+$ for which this mode of operation is maintained? For $V^+ = 2$ V find the values of $I$ and $V$. 

![Figure PD.4](image_url)

![Figure PD.6](image_url)