choose to stop at any midway point through the list of sections. When we teach the one-semester course, we often decide to go on to chapter 5 after covering section 4.6.

Chapters 5–7 are offered as sweet desserts. There are two distinct approaches to these last three chapters of the text. When we teach the course, we like to choose at least two or three sections from each of these chapters in order to give the students a taste of the many different disciplines in mathematics. Our students value this exposure—they say it helps them choose which courses they might later select from the upper-level offerings. Alternatively, each of the chapters is a completely independent module and can be studied in greater depth or omitted. Chapter 5 explores the mathematics of likelihood and the long-term patterns in discrete events. The section on hypothesis testing provides a mathematical approach to inductive thinking, parallel to the way in which chapter 1 provides a mathematical approach to deductive reasoning. The fundamental ideas introduced in this chapter include basic combinatorics, Pascal’s triangle, the binomial theorem, basic probability, hypothesis testing, and least squares regression. Many of the problems are computational, but the overriding framework of hypothesis testing and many of the abstract notions of probability theory are presented. This exposure is meant to assist greatly any student entering the corresponding upper level course.

Chapter 6 introduces the study of graphs by indicating how they model and solve real-world questions, beginning with the Königsberg bridge problem. In this way, the chapter describes the mathematics of adjacency and the abstract descriptions of networks of “connected” points (or objects). This chapter’s fundamental ideas include the definition and basic properties of graphs, Eulerian and Hamiltonian circuits, trees and spanning trees, and weighted graphs. The chapter presents many algorithms for constructing shortest paths, spanning trees, Hamiltonian cycles, and minimum weight versions of these objects in a given graph.

Chapter 7 presents an introduction to the theory of complex-valued functions, teaching students about the basic algebra of complex numbers, single- and multivalued functions such as $n$th roots, exponential, trigonometric, and logarithmic functions and their graphical representation, analytic functions, partial differentiation and the Cauchy–Riemann equations, power series representations of analytic functions, harmonic functions, and the Laplacian. An application section explores the use of streamlines and equipotentials to understand and model fluid flow.

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**Key Elements of the Text**

We hope *A Transition to Advanced Mathematics* will be recognized as a clear and cogent text in support of a transition course surveying mathematics. It is designed to serve ideally in collaboration with mathematics professors helping students to explore new mathematical vistas, to grow into the perspectives of the mathematician, and to successfully practice mathematics. The following elements of the text are intended to help facilitate this partnership between professor and text in the creation of a dynamic and interesting learning experience.
Embedded questions. In each section, after reading through the text and examples that illustrate and explain fundamental concepts, students are invited to create and display their personal understanding of the mathematical idea at hand by answering questions. Many of these queries are straightforward and useful in providing good introductory experiences with the new ideas at hand; as such, they can be assigned as homework in preparation for class or used during class in the spirit of active learning and engaged discussion. Some of them lead to a main idea of an upcoming proof. An example is question 3.1.9 in section 3.1, which asks students about computations of the form \( p_1 \cdot p_2 \cdots p_n + 1 \) where each \( p_k \) is prime—are integers of that form always prime? (It is still an open question whether there are infinitely many primes in this sequence.)

Reading questions. An effective pedagogical tool is to expect students to read the text before coming to class and to be able to answer a collection of basic questions. We always want our students to use a text more than as a reference for worked examples. Reading comprehension questions at the end of each section ask for definitions, examples, and the central ideas of the material, leading students to open the book and read.

There are many ways for an instructor to use the reading questions. We assign them before every class meeting and expect students to write their responses in complete English sentences. Our hope is that students both learn the value of reading the book and get practice in expressing mathematical concepts well. They also come better prepared for class. In this way, teachers can respond to students’ questions and engage the mathematical ideas at a much deeper level during class, and the students develop the independent reading skills essential for more sophisticated mathematical studies.

Exercises. Every section is accompanied by 70 exercises that allow the professor considerable flexibility in assigning homework and that give the reader practice. As with any exercise set, the ultimate goal is to provide students needed practice to deepen their understanding of the corresponding mathematical concepts. Instructors can pick and chose from many different types of problems. The exercises are grouped according to topic; if the instructor has focused on just part of the section’s material, it is easy to pick out corresponding problems to assign.

The end of each exercise set always contains a variety of more challenging exercises. These questions sometimes anticipate ideas in upcoming sections, require the study and use of a new definition or idea, or ask students to make conjectures based on some pattern arising from a collection of computations. Instructors could occasionally use them to motivate students to pursue a topic in more detail, or as a staging point for further investigations that might lead to a short paper or presentation.

An application section. Every chapter includes a section that explores an application of the theoretical ideas under study. All involve interesting “real-world” issues. Students are often surprised when theoretical notions find expression as a useful tool in life. The text intends to teach students, as they see a variety of applications, to view purely abstract, theoretical ideas as not antithetical to using mathematics to benefit society. The intent is for students to begin to perceive pure and applied mathematics as going hand in hand and strengthening one another in interplay: a search for applications often results in the development of new theoretical ideas, and theoretical mathematics often manifests itself as a critical underpinning of an applied tool.
None of the application sections are required for the text’s other sections. When we teach the course, we sometimes treat a chapter’s applied section in the same way as the others in the chapter, but at other times we might simply ask our students to read the section outside of class and submit the reading questions, or have them work in teams to answer some of the exercises. Depending upon the instructor’s interests and the parameters of the course, any applied section may be skipped.

Embedded reflections on the history, culture, and philosophy of mathematics. Mathematics is a timeless study that has been gradually developed through the corporate efforts of diverse individuals and cultures. The historical origins of mathematical ideas and the accompanying cultural standards for definitions, examples, and proofs are worthwhile and interesting and contribute to a student’s ability to understand and appreciate contemporary mathematics. Throughout the text, we tell stories about the struggles, the insights, and the people and events that helped shape mathematics. Our hope is for students to enjoy the drama, getting a sense of the eureka of mathematical breakthroughs and connecting proofs and mathematical statements (so often presented as devoid of human emotion), and relating to the human lives of the men and women who first presented them.

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